- Turn your test papers in on Tuesday, April 6.
- You must work individually on this test.
- Attach printouts of your computer code and its output with your comments.

Test 2

Problem 1. Describe an algorithm to solve a least squares problem with a full-rank matrix.

Problem 2. Write a MATLAB program that solves the least squares problem with a full-rank matrix $A \in R^{n\times m}$. Include a reasonable number of comments in your program.

The code should first implement a QR decomposition, $A = QR$, where $Q \in R^{n\times n}$ is a product of reflectors and $R = [\hat{R}\ 0]^T \in R^{n\times m}$, where $\hat{R}$ is $m \times m$ and upper triangular. Make use of algorithms (3.2.37), (3.2.40), and (3.2.45).

Then, the code should use the QR decomposition to find x, the solution of the least squares problem. Calculate $c = Q^T b = Q_m Q_{m-1} \dots Q_1 b$ by applying the reflectors subsequently. An additional one-dimensional array is needed for b. This array can also be used for c and intermediate results. The solution x is found by solving $\hat{R}x = \hat{c}$ by back substitution, where $c = [\hat{c}\ d]^T$. Find the minimum value of $\|Ax - b\|_2$ without computing $Ax - b$.

Problem 3.

1. Use your program to solve the following problems.
   (a) Find the least squares quadratic polynomial for the data.

t_i	-1	-0.75	-0.5	0	0.25	0.5	0.75
y_i	1	0.8125	0.75	1	1.3125	1.75	2.3125

   The exact solution is $\phi(t) = 1 + t + t^2$.

   (b) Using the basis $\phi_1(t) = 50$ $\phi_2(t) = t - 1065$, find the least squares linear polynomials for the following two sets of data:

t_i	1000	1050	1060	1080	1110	1130
(1) y_i	6010	6153	6421	6399	6726	6701
(2) y_i	9422	9300	9220	9150	9042	8800

   (Notice that the QR decomposition only needs to be done once.)

2. Solve the problems in (a) and (b) using MATLAB's "\" command.
3. Plot your solutions and data points for parts (a) and (b).

Problem 4. Let $A = (a_{ij})_{n\times n}$. Let $\|\cdot\|_\infty$ be a matrix norm induced by the maximum vector norm $\|x\|_\infty = \max_{1\le i\le n} |x_i|$, $x \in R^n$. Prove that

$$\|A\|_\infty = \max_{1\le i\le n} \sum_{j=1}^{n} |a_{ij}|.$$