Review Topics for Test II

Chapter 5. IVPs for ODEs

§5.2 Euler’s method. Formulation of Euler’s scheme, error estimate.

§5.3 Higher-Order Taylor Methods. Local truncation error, global error, Taylor method of order 2. The order of the local truncation error.


§5.5 Error control and Runge-Kutta-Fuhlberg method. The main idea on how the error control works. Why is Runge-Kutta-Fuhlberg scheme used for error control?

§5.6 Multistep methods. Definition of one-step methods, definition of an $m$-step method, explicit and implicit schemes, classification and local truncation errors of Adams-Bashforth and Adams-Moulton schemes, predictor-corrector schemes. How are schemes chosen for the predictor and the corrector steps?

§5.9 Higher-order equations and systems of DEs. Reduction of an IVP with a differential equation of order $n$ to the IVP of $n$-th order system of first order ODEs. Formulation of Runge-Kutta method of order 4 for a system of ODEs.

§5.11 Stiff DEs. What equations are classified as stiff? Definition of an absolutely stable scheme. Formulation of an implicit trapezoidal scheme. How are stiff problems solved?

Chapter 6. Direct methods for solving linear systems

§6.1 Linear systems of equations. Gaussian elimination algorithm (formulation, cost), forward and back substitution algorithms (formulations, costs)

§6.2 Pivoting strategies. What is a pivoting, and why is it used? Partial, scaled and complete pivoting strategies and their costs.


§6.6 Special types of matrices. Nonsingular matrix, strictly diagonally dominant matrix, symmetric matrix, positive definite matrix; relations between nonsingular, positive definite, and strictly diagonally dominant matrices; $LDL^T$ and Choleski’s factorizations, relation with positive definite matrices, cost, solving linear systems with such factorizations; band matrices, bandwidth, solution of a linear system with a tridiagonal matrix, cost.

Chapter 7. Iterative techniques in matrix algebra

§7.1 Norms of vectors and matrices. Vector norm, Euclidean (two) norm, maximum norm, distance between vectors, convergence of a sequence of vectors, matrix norms, natural or induced norms.
§7.3 Iterative techniques for solving linear systems. Direct and iterative methods, Jacobi and Gauss-Seidel methods, convergence of Jacobi and Gauss-Seidel, relaxation methods, SOR method, convergence of SOR.

Chapter 2. Solution of equations in one variable

§2.1 The bisection method. Formulation and convergence.

§2.2 Fixed-point iterations. Definition of a fixed point, existence and uniqueness of a fixed point, fixed-point iteration, convergence.

§2.3 Newton’s method. Formulation, convergence, secant method.

Chapter 10. Numerical solution of nonlinear systems of equations

§10.1 Fixed points for functions of several variables. Definition of a fixed point, continuous and Lipschitz continuous vector functions, existence of a fixed point, uniqueness of a fixed point, fixed point iteration, convergence; linear, superlinear, superlinear of order $\alpha$, and quadratic convergences.

§10.2 Newton’s method. Jacobian, formulation and implementation of Newton’s method, convergence.