

Problem	1	2	3	4	Grade
Points	/10	/10	/10	/10	/40

Name: _____

Test 3

Show all your work. Do not use a calculator nor any crib notes.

Problem 1 (10 pts) Solve the initial value problem

$$\mathbf{x}' = \begin{pmatrix} -3 & 2 \\ -3 & 4 \end{pmatrix} \mathbf{x}, \quad \mathbf{x}(0) = \begin{pmatrix} 0 \\ -5 \end{pmatrix}.$$

Problem 2 (10 pts) Find the general solution of the system of differential equations

$$\mathbf{x}' = \begin{pmatrix} -3 & 1 \\ -1 & -1 \end{pmatrix} \mathbf{x}.$$

Problem 3 (10 pts) Let

$$A = \begin{pmatrix} 2 & -1 \\ 1 & 2 \end{pmatrix} \quad \text{and} \quad \mathbf{f} = e^{2t} \begin{pmatrix} 1 \\ 2 \end{pmatrix}.$$

A system of differential equations $\mathbf{x}' = A\mathbf{x}$ has a general solution

$$\mathbf{x}_h(t) = c_1 e^{2t} \begin{pmatrix} -\sin t \\ \cos t \end{pmatrix} + c_2 e^{2t} \begin{pmatrix} \cos t \\ \sin t \end{pmatrix}.$$

Find a particular solution of the nonhomogeneous system $\mathbf{x}' = A\mathbf{x} + \mathbf{f}$.

Problem 4 (10 pts) Identify the type and stability of the critical point $(0, 0)$ of a linear system $\mathbf{x}' = A\mathbf{x}$ where

- (see Problem 1)

$$A = \begin{pmatrix} -3 & 2 \\ -3 & 4 \end{pmatrix}$$

- (see Problem 2)

$$A = \begin{pmatrix} -3 & 1 \\ -1 & -1 \end{pmatrix}$$

- (see Problem 3)

$$A = \begin{pmatrix} 2 & -1 \\ 1 & 2 \end{pmatrix}$$