

Problem	1	2	3	4	5	Grade
Points	/20	/20	/20	/20	/20	/100

Name: _____

Test 2

Show all your work. Do not use a calculator nor any crib notes.

Problem 1 (20 pts) Let

$$f(t) = \begin{cases} 6t - 9, & 0 \leq t < 3 \\ t^2, & t \geq 3 \end{cases}$$

1. Write $f(t)$ in terms of the unit step function.
2. Find the Laplace transform of $f(t)$.

Problem 2 (20 pts) A weight of 1 *lb* stretches a spring 2 *in*. The whole spring-mass system is immersed into a fluid which offers a resistive force equal to the half of the instantaneous velocity. At the initial time, the weight is released 2 *in* below the equilibrium position with the upward initial velocity 10 *ft/sec*.

1. Set up the initial value problem for the displacement $x(t)$ of the weight.
2. Determine the type of this motion:

Underdamped Critically damped Overdamped

Problem 3 (20 pts) Solve the initial value problem using the Laplace transform.

$$y'' - 2y' - 8y = 0, \quad t > 0, \quad y(0) = 0, \quad y'(0) = -1.$$

Problem 4 (20 pts) Using the method of variation of parameters, find the general solution of the differential equation

$$y'' - 6y' + 9y = x^{-4}e^{3x}, \quad x > 0.$$

Problem 5 (20 pts) Compute the inverse Laplace transform of

$$\frac{e^{-s}}{s^2 + 6s + 13}$$