Problem 5. Let $A = \begin{pmatrix} 1 & -2 & 6 & 5 \\ 0 & 1 & -1 & -3 \\ 0 & -1 & 2 & 3 \\ 1 & 1 & -1 & -4 \end{pmatrix}$.

1. Find a basis for the null space of matrix A. Solve $Ax=0$.

$$\begin{pmatrix} 1 & -2 & 6 & 5 \\ 0 & 1 & -1 & -3 \\ 0 & -1 & 2 & 3 \\ 1 & 1 & -1 & -4 \end{pmatrix} \to \begin{pmatrix} 1 & -2 & 6 & 5 \\ 0 & 1 & -1 & -3 \\ 0 & -1 & 2 & 3 \\ 0 & 3 & -7 & -9 \end{pmatrix}_{R_4-R_1} \to \begin{pmatrix} 1 & -2 & 6 & 5 \\ 0 & 1 & -1 & -3 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & -4 & 0 \end{pmatrix}_{R_4-3R_2}$$

$$\to \begin{pmatrix} 1 & -2 & 6 & 5 \\ 0 & 1 & -1 & -3 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}_{R_4+4R_3} \to \begin{pmatrix} 1 & -2 & 0 & 5 \\ 0 & 1 & 0 & -3 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix} \to \begin{pmatrix} 1 & 0 & 0 & -1 \\ 0 & 1 & 0 & -3 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}$$

$$\begin{cases} x_4 = t \\ x_3 = 0 \\ x_2 = 3t \\ x_1 = t \end{cases} \Rightarrow \quad x = t\begin{pmatrix} 1 \\ 3 \\ 0 \\ 1 \end{pmatrix}.$$

Basis for $N(A)$ is

$$B = \left\{ \begin{pmatrix} 1 \\ 3 \\ 0 \\ 1 \end{pmatrix} \right\}.$$

2. Find a basis for the column space of matrix A.

Columns 1, 2, and 3 are pivot.
Basis for $\mathrm{col}(A)$ is

$$\left\{ \begin{pmatrix} 1 \\ 0 \\ 0 \\ 1 \end{pmatrix}, \begin{pmatrix} -2 \\ 1 \\ -1 \\ 1 \end{pmatrix}, \begin{pmatrix} 6 \\ -1 \\ 2 \\ -1 \end{pmatrix} \right\}.$$

3. Find a basis for the row space of matrix A.

$$\{ (1,0,0,-1), (0,1,0,-3), (0,0,1,0) \}.$$

4. Specify the rank of matrix A. $\mathrm{rank}(A) = 3 = \#$ of pivots.