Problem 3. The set $B = \{1-t-t^2,\ t-t^2,\ 2-2t+t^2\}$ is a basis for P_2. Find the B-coordinat
vector of $p(t) = 5-7t$.

Let $p_1 = 1-t-t^2$, $p_2 = t-t^2$, $p_3 = 2-2t+t^2$.

Let $E = \{1, t, t^2\}$ be the standard basis for P_2.

$$[p_1]_E = \begin{pmatrix} 1 \\ -1 \\ -1 \end{pmatrix}, \quad [p_2]_E = \begin{pmatrix} 0 \\ 1 \\ -1 \end{pmatrix}, \quad [p_3] = \begin{pmatrix} 2 \\ -2 \\ 1 \end{pmatrix}.$$

$$P_{E\leftarrow B} = \begin{pmatrix} 1 & 0 & 2 \\ -1 & 1 & -2 \\ -1 & -1 & 1 \end{pmatrix}. \quad [p]_E = \begin{pmatrix} 5 \\ -7 \\ 0 \end{pmatrix}.$$

$[p]_E = P_{E\leftarrow B}[p]_B \Leftarrow$ solve.

$$[P \mid [p]_E] = \left(\begin{array}{ccc|c} 1 & 0 & 2 & 5 \\ -1 & 1 & -2 & -7 \\ -1 & -1 & 1 & 0 \end{array}\right) \to \begin{pmatrix} 1 & 0 & 2 & 5 \\ 0 & 1 & 0 & -2 \\ 0 & -1 & 3 & 5 \end{pmatrix} \begin{matrix} \\ R_2+R_1 \\ R_3+R_1 \end{matrix}$$

$$\to \begin{pmatrix} 1 & 0 & 2 & 5 \\ 0 & 1 & 0 & -2 \\ 0 & 0 & 3 & 3 \end{pmatrix} R_3+R_2 \to \begin{pmatrix} 1 & 0 & 2 & 5 \\ 0 & 1 & 0 & -2 \\ 0 & 0 & 1 & 1 \end{pmatrix} \tfrac{1}{3}R_3$$

$$\to \left(\begin{array}{ccc|c} 1 & 0 & 0 & 3 \\ 0 & 1 & 0 & -2 \\ 0 & 0 & 1 & 1 \end{array}\right) \Rightarrow [p]_B = \begin{pmatrix} 3 \\ -2 \\ 1 \end{pmatrix}.$$

<u>Verify:</u>

$$3\cdot p_1 - 2p_2 + p_3 = 3(1-t-t^2) - 2(t-t^2) + (2-2t+t^2)$$

$$= 5 - 7t = p(t). \quad \text{Correct.}$$