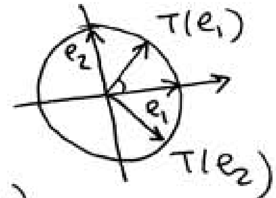
Problem 4. 1. Find the matrix of the linear transformation $T: R^2 \to R^2$ on the plane that first, reflects through the line $x_2 = 0$, then rotates about the origin counterclockwise through the angle $\pi/4$.

Find images of $e_1 = \begin{pmatrix} 1 \\ 0 \end{pmatrix}$ and $e_2 = \begin{pmatrix} 0 \\ 1 \end{pmatrix}$.

$$e_1 \xrightarrow{\text{Reflect}} e_1 \xrightarrow{\text{Rotate}} \begin{pmatrix} 1/\sqrt{2} \\ 1/\sqrt{2} \end{pmatrix} = T(e_1)$$

$$e_2 \to -e_2 \to \begin{pmatrix} 1/\sqrt{2} \\ -1/\sqrt{2} \end{pmatrix} = T(e_2)$$



$$[T] = [T(e_1), T(e_2)] = \begin{bmatrix} \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} \end{bmatrix} = \frac{1}{\sqrt{2}} \begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix}.$$

2. Find the image of point $\mathbf{p} = (7, -3)$ under the transformation T.

$$T(p) = [T]\,p = \frac{1}{\sqrt{2}} \begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix} \begin{bmatrix} 7 \\ -3 \end{bmatrix} = \frac{1}{\sqrt{2}} \begin{bmatrix} 7-3 \\ 7+3 \end{bmatrix} = \frac{1}{\sqrt{2}} \begin{bmatrix} 4 \\ 10 \end{bmatrix}$$

$$T(p) = \sqrt{2} \begin{bmatrix} 2 \\ 5 \end{bmatrix}.$$