

Problem 3. Let $T(x) = Ax$, where

$$A = \begin{pmatrix} 1 & 3 & -2 \\ -3 & 4 & 1 \\ 2 & 4 & 2 \\ -3 & 1 & 3 \end{pmatrix}.$$

Explain your answers.

1. Specify the domain and the co-domain of T .

Domain: \mathbb{R}^3

Co-domain: \mathbb{R}^4

2. Is T one-to-one?

Find pivot positions of A .

$$\begin{bmatrix} 1 & 3 & -2 \\ -3 & 4 & 1 \\ 2 & 4 & 2 \\ -3 & 1 & 3 \end{bmatrix} \rightarrow \begin{bmatrix} 1 & 3 & -2 \\ 0 & 13 & -5 \\ 0 & -2 & 6 \\ 0 & 10 & -3 \end{bmatrix} \begin{array}{l} R_2 + 3R_1 \\ R_3 - 2R_1 \\ R_4 + 3R_1 \end{array} \rightarrow \begin{bmatrix} 1 & 3 & -2 \\ 0 & 1 & -3 \\ 0 & 13 & -5 \\ 0 & 10 & -3 \end{bmatrix} \begin{array}{l} R_1 \\ -\frac{1}{2}R_3 \\ R_2 \\ R_4 \end{array}$$

$$\rightarrow \begin{bmatrix} 1 & 3 & -2 \\ 0 & 1 & -3 \\ 0 & 0 & 34 \\ 0 & 0 & 27 \end{bmatrix} \begin{array}{l} R_1 \\ R_2 \\ R_3 - 13R_2 \\ R_4 - 10R_2 \end{array} \rightarrow \begin{bmatrix} 1 & 3 & -2 \\ 0 & 1 & -3 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{bmatrix} \begin{array}{l} R_1 \\ R_2 \\ \frac{1}{34}R_3 \end{array}$$

Every column of A has a pivot position.

$\Rightarrow Ax=0$ has only the trivial solution.

$\Rightarrow T(x)=0$ has only the zero solution.

\Rightarrow T is one-to-one.

3. Is T onto?

No, T is not. A doesn't have a pivot in every row.
The equation $Ax=b$ may be inconsistent for some b .

4. Is T invertible?

No, T is Not onto. $\Rightarrow T$ is not invertible.