

Test 1

Problem	1	2	3	4	5	Grade
Points	/10	/10	/10	/10	/10	/50

NAME: Solution Key

Show all your work for a full credit. Crib sheets and notes are not allowed.

Problem 1. Find the second column of the inverse of matrix A without computing all entries of the

$$\text{inverse: } A = \begin{pmatrix} 2 & 3 & -1 \\ 1 & 1 & 0 \\ 1 & -2 & 2 \end{pmatrix}.$$

Let \bar{c}_2 be the second column of A^{-1} . Then, $A\bar{c}_2 = \bar{e}_2$, where $\bar{e}_2 = \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix}$. Solving thissystem, we find \bar{c}_2 . Row reduce $[A|\bar{e}_2]$

$$\begin{bmatrix} 2 & 3 & -1 & 0 \\ 1 & 1 & 0 & 1 \\ 1 & -2 & 2 & 0 \end{bmatrix} \rightarrow \begin{bmatrix} 1 & 1 & 0 & 1 \\ 2 & 3 & -1 & 0 \\ 1 & -2 & 2 & 0 \end{bmatrix} \begin{matrix} R_2 \\ R_1 \\ R_3 \end{matrix} \rightarrow \begin{bmatrix} 1 & 1 & 0 & 1 \\ 0 & 1 & -1 & -2 \\ 0 & -3 & 2 & -1 \end{bmatrix} \begin{matrix} R_2 - 2R_1 \\ R_3 - R_1 \end{matrix}$$

$$\rightarrow \begin{bmatrix} 1 & 1 & 0 & 1 \\ 0 & 1 & -1 & -2 \\ 0 & 0 & -1 & -7 \end{bmatrix} \begin{matrix} R_1 \\ R_2 \\ R_3 + 3R_2 \end{matrix} \rightarrow \begin{bmatrix} 1 & 1 & 0 & 1 \\ 0 & 1 & -1 & -2 \\ 0 & 0 & 1 & 7 \end{bmatrix}$$

$$\rightarrow \begin{bmatrix} 1 & 1 & 0 & 1 \\ 0 & 1 & 0 & 5 \\ 0 & 0 & 1 & 7 \end{bmatrix} \begin{matrix} R_2 + R_3 \\ R_1 - R_2 \end{matrix} \rightarrow \begin{bmatrix} 1 & 0 & 0 & -4 \\ 0 & 1 & 0 & 5 \\ 0 & 0 & 1 & 7 \end{bmatrix}$$

$$\Rightarrow \bar{c}_2 = \begin{bmatrix} -4 \\ 5 \\ 7 \end{bmatrix}.$$