

Problem 5. Let

$$S = \left\{ \begin{bmatrix} 2 \\ 1 \\ 2 \end{bmatrix}, \begin{bmatrix} 3 \\ 1 \\ 2 \end{bmatrix}, \begin{bmatrix} 2 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} -2 \\ 2 \\ -3 \end{bmatrix} \right\} \text{ and } \mathbf{b} = \begin{bmatrix} 0 \\ 8 \\ -5 \end{bmatrix}.$$

1. Is the set S linearly dependent or independent? Explain.

S is linearly dependent because it consists of 4 3-D vectors.

2. Find a linearly independent subset of the set S that generates the span of S .

$$A = \begin{pmatrix} 2 & 3 & 2 & -2 \\ 1 & 1 & 0 & 2 \\ 2 & 2 & 0 & -3 \end{pmatrix} \rightarrow \begin{pmatrix} 1 & 1 & 0 & 2 \\ 2 & 3 & 2 & -2 \\ 2 & 2 & 0 & -3 \end{pmatrix} \rightarrow \begin{pmatrix} 1 & 1 & 0 & 2 \\ 0 & 1 & 2 & -6 \\ 0 & 0 & 0 & -7 \end{pmatrix}$$

$\rightarrow \begin{pmatrix} 1 & 1 & 0 & 2 \\ 0 & 1 & 2 & -6 \\ 0 & 0 & 0 & -7 \end{pmatrix}$ 1st, 2nd, and 4th columns are pivot

\Rightarrow The set $\left\{ \begin{pmatrix} 2 \\ 1 \\ 2 \end{pmatrix}, \begin{pmatrix} 3 \\ 1 \\ 2 \end{pmatrix}, \begin{pmatrix} -2 \\ 2 \\ -3 \end{pmatrix} \right\}$

is lin. independent and spans $\text{span}(S)$.

3. Determine if vector \mathbf{b} belongs to the span of vectors in S ? If yes, then express \mathbf{b} as a linear combination of vectors in S .

$$\left(\begin{array}{ccc|c} 2 & 3 & -2 & 0 \\ 1 & 1 & 2 & 8 \\ 2 & 2 & -3 & -5 \end{array} \right) \rightarrow \left(\begin{array}{ccc|c} 1 & 1 & 2 & 8 \\ 2 & 3 & -2 & 0 \\ 2 & 2 & -3 & -5 \end{array} \right) \rightarrow \left(\begin{array}{ccc|c} 1 & 1 & 2 & 8 \\ 0 & 1 & -6 & -16 \\ 0 & 0 & -7 & -21 \end{array} \right) \begin{array}{l} \text{No row } [0,0,0,*] \\ \Rightarrow \mathbf{b} \in \text{span}(S). \end{array}$$

$$\rightarrow \left(\begin{array}{ccc|c} 1 & 1 & 2 & 8 \\ 0 & 1 & -6 & -16 \\ 0 & 0 & 1 & 3 \end{array} \right) \rightarrow \left(\begin{array}{ccc|c} 1 & 1 & 0 & 2 \\ 0 & 1 & 0 & 2 \\ 0 & 0 & 1 & 3 \end{array} \right) \rightarrow \left(\begin{array}{ccc|c} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 2 \\ 0 & 0 & 1 & 3 \end{array} \right)$$

$\Rightarrow x_1=0, x_2=2, x_3=3$

$\Rightarrow \mathbf{b} = \begin{pmatrix} 0 \\ 8 \\ -5 \end{pmatrix} = 2 \begin{pmatrix} 3 \\ 1 \\ 2 \end{pmatrix} + 3 \begin{pmatrix} -2 \\ 2 \\ -3 \end{pmatrix}.$

4. Does the set S span \mathbb{R}^3 ? Explain.

Yes. Matrix A has a pivot in every row.