

**Problem 4.** Let  $A = \begin{pmatrix} 3 & 0 \\ 1 & 3 \end{pmatrix}$  and  $B = \begin{pmatrix} 2 & -1 \\ 5 & 3 \end{pmatrix}$ . Find the product  $AB$  using:

1. The row-column rule;

$$AB = \begin{pmatrix} 3 \cdot 2 + 0 \cdot 5 & 3(-1) + 0 \cdot 3 \\ 1 \cdot 2 + 3 \cdot 5 & 1 \cdot (-1) + 3 \cdot 3 \end{pmatrix} = \begin{pmatrix} 6 & -3 \\ 17 & 8 \end{pmatrix}.$$

2. The column-row rule;

$$\begin{aligned} AB &= \begin{pmatrix} 3 \\ 1 \end{pmatrix} (2, -1) + \begin{pmatrix} 0 \\ 3 \end{pmatrix} (5, 3) \\ &= \begin{pmatrix} 6 & -3 \\ 2 & -1 \end{pmatrix} + \begin{pmatrix} 0 & 0 \\ 15 & 9 \end{pmatrix} = \begin{pmatrix} 6 & -3 \\ 17 & 8 \end{pmatrix} \end{aligned}$$

3. The matrix-vector product rule.

$$\begin{aligned} AB &= (AB_1, AB_2) = \left( \begin{pmatrix} 3 & 0 \\ 1 & 3 \end{pmatrix} \begin{pmatrix} 2 \\ 5 \end{pmatrix}, \begin{pmatrix} 3 & 0 \\ 1 & 3 \end{pmatrix} \begin{pmatrix} -1 \\ 3 \end{pmatrix} \right) \\ &= \begin{pmatrix} 6+0 & -3+0 \\ 2+15 & -1+9 \end{pmatrix} = \begin{pmatrix} 6 & -3 \\ 17 & 8 \end{pmatrix}. \end{aligned}$$