Problem 3. Let $T(x) = Ax$ for $x \in R^3$, where

$$A = \begin{pmatrix} 2 & -1 & 3 \\ -1 & 0 & 2 \\ -1 & -1 & 9 \\ 1 & 2 & 0 \end{pmatrix}.$$

Explain your answers.

1. Show that T is a linear transformation.

$T(\bar{x}_1 + \bar{x}_2) = A(\bar{x}_1 + \bar{x}_2) = A\bar{x}_1 + A\bar{x}_2, \ \forall \bar{x}_1, \bar{x}_2 \in R^3.$

$T(c\bar{x}) = A(c\bar{x}) = cA\bar{x} = cT(x), \ \forall \bar{x} \in R^3, \forall c \in R.$

$\Rightarrow$ T is a linear transformation.

2. Specify the domain and the co-domain of T.

Domain of T is R^3.

Co-domain of T is R^4.

$T: R^3 \to R^4$.

3. Is T one-to-one?

$$\begin{pmatrix} 2 & -1 & 3 \\ -1 & 0 & 2 \\ -1 & -1 & 9 \\ 1 & 2 & 0 \end{pmatrix} \to \begin{pmatrix} 1 & 2 & 0 \\ -1 & 0 & 2 \\ -1 & -1 & 9 \\ 2 & -1 & 3 \end{pmatrix} \to \begin{pmatrix} 1 & 2 & 0 \\ 0 & 2 & 2 \\ 0 & 1 & 9 \\ 0 & -5 & 3 \end{pmatrix} \to \begin{pmatrix} 1 & 2 & 0 \\ 0 & 1 & 1 \\ 0 & 1 & 9 \\ 0 & -5 & 3 \end{pmatrix}$$

$$\to \begin{pmatrix} 1 & 2 & 0 \\ 0 & 1 & 1 \\ 0 & 0 & 1 \\ 0 & 0 & 1 \end{pmatrix} \to \begin{pmatrix} 1 & 2 & 0 \\ 0 & 1 & 1 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{pmatrix}$$

Pivot in every column

T is one-to-one.

4. Is T onto?

T is not onto since columns of A don't span R^4. The forth row of A doesn't have a pivot position

5. Is T invertible? No. T is invertible $\Leftrightarrow$ A is invertible. A isn't invertible since it isn't a square matrix