

Problem 2. Let  $A = \begin{pmatrix} -2 & 1 & 0 & 0 & 0 \\ 8 & -3 & 0 & 0 & 0 \\ 0 & 0 & 1 & 2 & 0 \\ 0 & 0 & 3 & 4 & 0 \\ 0 & 0 & 0 & 0 & 7 \end{pmatrix}$ .  $A = \begin{pmatrix} A_1 & & \\ & A_2 & \\ & & A_3 \end{pmatrix}$ ,  $A^{-1} = \begin{pmatrix} A_1^{-1} & & \\ & A_2^{-1} & \\ & & A_3^{-1} \end{pmatrix}$

1. Find  $A^{-1}$  without row reducing matrix  $A$ .

$$A_1 = \begin{pmatrix} -2 & 1 \\ 8 & -3 \end{pmatrix} \Rightarrow \bar{A}_1^{-1} = \frac{1}{6-8} \begin{pmatrix} -3 & -1 \\ -8 & -2 \end{pmatrix} = \begin{pmatrix} \frac{3}{2} & \frac{1}{2} \\ 4 & 1 \end{pmatrix}$$

$$A_2 = \begin{pmatrix} 1 & 2 \\ 3 & 4 \end{pmatrix} \Rightarrow \bar{A}_2^{-1} = \frac{1}{4-6} \begin{pmatrix} 4 & -2 \\ -3 & 1 \end{pmatrix} = \begin{pmatrix} -2 & 1 \\ \frac{3}{2} & -\frac{1}{2} \end{pmatrix}$$

$$A_3 = (7) \Rightarrow \bar{A}_3^{-1} = \left(\frac{1}{7}\right)$$

$$A^{-1} = \begin{pmatrix} \frac{3}{2} & \frac{1}{2} & & & \\ 4 & 1 & & & \\ & & -2 & 1 & \\ & & \frac{3}{2} & -\frac{1}{2} & \\ & & & & \frac{1}{7} \end{pmatrix}$$

All unspecified entries are zero.

2. Using  $A^{-1}$ , find the inverse of  $A^T$ .

$$(A^T)^{-1} = (A^{-1})^T = \begin{pmatrix} \frac{3}{2} & 4 & & & \\ \frac{1}{2} & 1 & & & \\ & & -2 & 1 & \\ & & \frac{3}{2} & -\frac{1}{2} & \\ & & & & \frac{1}{7} \end{pmatrix}$$