Test 1

Problem	1	2	3	4	5	Grade
Points	/10	/10	/10	/10	/10	/50

NAME: Solution Key

Show all your work for a full credit. Calculators and crib sheets are not allowed.

Problem 1. Let $A = \begin{pmatrix} 1 & -1 & 0 & 3 & 1 \\ 1 & 1 & 2 & 1 & -1 \\ 2 & -1 & 2 & 8 & 5 \end{pmatrix}$ and $b = \begin{pmatrix} 2 \\ 4 \\ 4 \end{pmatrix}$.

1. Solve the linear system $Ax = b$ and write the solution in the parametric vector form.

$$(A|b) = \begin{pmatrix} 1 & -1 & 0 & 3 & 1 & 2 \\ 1 & 1 & 2 & 1 & -1 & 4 \\ 2 & -1 & 2 & 8 & 5 & 4 \end{pmatrix} \to \begin{pmatrix} 1 & -1 & 0 & 3 & 1 & 2 \\ 0 & 2 & 2 & -2 & -2 & 2 \\ 0 & 1 & 2 & 2 & 3 & 0 \end{pmatrix}$$

$$\to \begin{pmatrix} 1 & -1 & 0 & 3 & 1 & 2 \\ 0 & 1 & 1 & -1 & -1 & 1 \\ 0 & 1 & 2 & 2 & 3 & 0 \end{pmatrix} \to \begin{pmatrix} 1 & -1 & 0 & 3 & 1 & 2 \\ 0 & 1 & 1 & -1 & -1 & 1 \\ 0 & 0 & 1 & 3 & 4 & -1 \end{pmatrix}$$

linear system is consistent (No row as $[0,0,0,0,0,*]$)

$$\to \begin{pmatrix} 1 & -1 & 0 & 3 & 1 & 2 \\ 0 & 1 & 0 & -4 & -5 & 2 \\ 0 & 0 & 1 & 3 & 4 & -1 \end{pmatrix} \to \left(\begin{array}{ccccc|c} 1 & 0 & 0 & -1 & -4 & 4 \\ 0 & 1 & 0 & -4 & -5 & 2 \\ 0 & 0 & 1 & 3 & 4 & -1 \end{array}\right)$$

x_1, x_2, x_3 – basic variables; x_4, x_5 – free variables.

$$\begin{cases} x_5 = t \\ x_4 = s \\ x_3 = -1 - 3s - 4t \\ x_2 = 2 + 4s + 5t \\ x_1 = 4 + s + 4t \end{cases} \Rightarrow \bar{x} = \begin{pmatrix} 4 \\ 2 \\ -1 \\ 0 \\ 0 \end{pmatrix} + s \begin{pmatrix} 1 \\ 4 \\ -3 \\ 1 \\ 0 \end{pmatrix} + t \begin{pmatrix} 4 \\ 5 \\ -4 \\ 0 \\ 1 \end{pmatrix}, \quad s, t \in R.$$

2. Find a general solution of the corresponding homogeneous linear system.

$$\bar{x} = s \begin{pmatrix} 1 \\ 4 \\ -3 \\ 1 \\ 0 \end{pmatrix} + t \begin{pmatrix} 4 \\ 5 \\ -4 \\ 0 \\ 1 \end{pmatrix}.$$

3. Does the linear system have a solution for any $b \in R^3$? Explain.

Yes. Columns of A span R^3 since every row has a pivot position.