Problem 4. Find an orthogonal set that generates span(S), where

$$S = \left\{ \mathbf{v}_1 = \begin{pmatrix} 1 \\ 1 \\ 1 \\ 1 \end{pmatrix}, \ \mathbf{v}_2 = \begin{pmatrix} 1 \\ 1 \\ 2 \\ 4 \end{pmatrix}, \ \mathbf{v}_3 = \begin{pmatrix} 1 \\ 2 \\ -4 \\ -3 \end{pmatrix} \right\}.$$

Apply the Gram-Schmidt process on S.

$$w_1 = v_1 = \begin{pmatrix} 1 \\ 1 \\ 1 \\ 1 \end{pmatrix}. \quad \|w_1\|^2 = 1+1+1+1 = 4$$

$$w_2 = v_2 - \frac{v_2 \cdot w_1}{\|w_1\|^2} w_1 = \begin{pmatrix} 1 \\ 1 \\ 2 \\ 4 \end{pmatrix} - \frac{1+1+2+4}{4} \begin{pmatrix} 1 \\ 1 \\ 1 \\ 1 \end{pmatrix} = \begin{pmatrix} -1 \\ -1 \\ 0 \\ 2 \end{pmatrix}$$

$$w_3 = v_3 - \frac{v_3 \cdot w_1}{\|w_1\|^2} w_1 - \frac{v_3 \cdot w_2}{\|w_2\|^2} w_2$$

$$= \begin{pmatrix} 1 \\ 2 \\ -4 \\ -3 \end{pmatrix} - \frac{\cancel{1}+\cancel{2}-4-\cancel{3}}{4} \begin{pmatrix} 1 \\ 1 \\ 1 \\ 1 \end{pmatrix} - \frac{-1-2-6}{1+1+4} \begin{pmatrix} -1 \\ -1 \\ 0 \\ 2 \end{pmatrix}$$

$$= \begin{pmatrix} 1 \\ 2 \\ -4 \\ -3 \end{pmatrix} + \begin{pmatrix} 1 \\ 1 \\ 1 \\ 1 \end{pmatrix} + \frac{3}{2} \begin{pmatrix} -1 \\ -1 \\ 0 \\ 2 \end{pmatrix} = \begin{pmatrix} 2 \\ 3 \\ -3 \\ -2 \end{pmatrix} + \frac{3}{2} \begin{pmatrix} -1 \\ -1 \\ 0 \\ 2 \end{pmatrix}$$

$$= \frac{1}{2} \begin{pmatrix} 4-3 \\ 6-3 \\ -6+0 \\ -4+6 \end{pmatrix} = \frac{1}{2} \begin{pmatrix} 1 \\ 3 \\ -6 \\ 2 \end{pmatrix} = \begin{pmatrix} 1/2 \\ 3/2 \\ -3 \\ 1 \end{pmatrix}.$$

$$\left\{ \begin{pmatrix} 1 \\ 1 \\ 1 \\ 1 \end{pmatrix}, \begin{pmatrix} -1 \\ -1 \\ 0 \\ 2 \end{pmatrix}, \begin{pmatrix} 1 \\ 3 \\ -6 \\ 2 \end{pmatrix} \right\}.$$