Problem 3. Find the orthogonal projection of vector $\mathbf{a} = (1, 2, -4, 2)^T$ on the subspace of R^4 spanned by the vectors $\mathbf{v}_1 = (1, 2, 1, 2)^T$ and $\mathbf{v}_2 = (1, 0, 1, -1)^T$.

Let $\hat{a} \in \operatorname{span}\{\bar{v}_1, \bar{v}_2\}$ be the orthogonal projection of vector $\bar{a}$ on $W = \operatorname{span}\{\bar{v}_1, \bar{v}_2\}$.

$$\hat{a} = \frac{\bar{a}\cdot\bar{v}_1}{\|\bar{v}_1\|^2} v_1 + \frac{\bar{a}\cdot\bar{v}_2}{\|\bar{v}_2\|^2} v_2 \quad \text{if } \bar{v}_1 \perp v_2. \quad \text{Note } \bar{v}_1\cdot\bar{v}_2 = 1+1-2=0.$$

$$\Rightarrow \bar{v}_1 \perp \bar{v}_2.$$

$$\bar{a}\cdot\bar{v}_1 = \begin{pmatrix} 1 \\ 2 \\ -4 \\ 2 \end{pmatrix}\cdot\begin{pmatrix} 1 \\ 2 \\ 1 \\ 2 \end{pmatrix} = 1+4-4+4 = 5$$

$$\|\bar{v}_1\|^2 = 1+4+1+4 = 10$$

$$\bar{a}\cdot\bar{v}_2 = \begin{pmatrix} 1 \\ 2 \\ -4 \\ 2 \end{pmatrix}\cdot\begin{pmatrix} 1 \\ 0 \\ 1 \\ -1 \end{pmatrix} = 1-4-2 = -5$$

$$\|\bar{v}_2\|^2 = 1+1+1 = 3$$

$$\hat{a} = \frac{5}{10} v_1 - \frac{5}{3} v_2 = \frac{5}{30}(3v_1 - 10v_2) =$$

$$= \frac{1}{6}\begin{pmatrix} 3-10 \\ 6-0 \\ 3-10 \\ 6+10 \end{pmatrix} = \frac{1}{6}\begin{pmatrix} -7 \\ 6 \\ -7 \\ 16 \end{pmatrix} = \begin{pmatrix} -7/6 \\ 1 \\ -7/6 \\ 8/3 \end{pmatrix}.$$