

Test 3

| Problem | 1 | 2 | 3 | 4 | 5 | Score |
|---------|---|---|---|---|---|-------|
| Points | | | | | | |

NAME: Solution key

Show all your work for full credit.

Problem 1. Let $W = \text{span}(S)$, where

$$S = \{(1, 2, 3, -1, 2)^T, (2, 4, 7, 2, -1)^T\}.$$

Find a basis for the orthogonal complement W^\perp of W . (Hint: use the fact $\text{Col}(A)^\perp = \text{Null}(A^T)$ for any matrix A .)

Let $A = \begin{pmatrix} 1 & 2 \\ 2 & 4 \\ 3 & 7 \\ -1 & 2 \\ 2 & -1 \end{pmatrix}$. Then $W = \text{Col}(A)$.

$$W^\perp = \text{Col}(A)^\perp = \text{Null}(A^T) \Rightarrow \text{Find a basis for Null}(A^T).$$

Solve $A^T x = 0$.

$$\begin{pmatrix} 1 & 2 & 3 & -1 & 2 \\ 2 & 4 & 7 & 2 & -1 \end{pmatrix} \rightarrow \begin{pmatrix} 1 & 2 & 3 & -1 & 2 \\ 0 & 0 & 1 & 4 & -5 \end{pmatrix} \rightarrow \begin{pmatrix} 1 & 2 & 0 & -13 & 17 \\ 0 & 0 & 1 & 4 & -5 \end{pmatrix}.$$

Basic variables: x_1, x_3 .Free variables: x_2, x_4, x_5

$$\begin{cases} x_5 = u \\ x_4 = t \\ x_3 = -4x_4 + 5x_5 = -4t + 5u \end{cases}$$

$$\begin{cases} x_2 = s \\ x_1 = -2x_2 + 13x_4 - 17x_5 = -2s + 13t - 17u \end{cases}$$

$$\Rightarrow \bar{x} = s \begin{pmatrix} -2 \\ 1 \\ 0 \\ 0 \\ 0 \end{pmatrix} + t \begin{pmatrix} 13 \\ 0 \\ -4 \\ 1 \\ 0 \end{pmatrix} + u \begin{pmatrix} -17 \\ 0 \\ 5 \\ 0 \\ 1 \end{pmatrix}$$

$$\forall s, t, u \in \mathbb{R}.$$

The set $\beta = \left\{ \begin{pmatrix} -2 \\ 1 \\ 0 \\ 0 \\ 0 \end{pmatrix}, \begin{pmatrix} 13 \\ 0 \\ -4 \\ 1 \\ 0 \end{pmatrix}, \begin{pmatrix} -17 \\ 0 \\ 5 \\ 0 \\ 1 \end{pmatrix} \right\}$ is a basis for W^\perp .