

Problem 6. Find the eigenvalues of the matrix and specify their algebraic multiplicities:

$$A = \begin{pmatrix} -4 & 0 & -3 \\ 1 & 2 & 5 \\ 0 & 0 & -4 \end{pmatrix}$$

$$p_A(t) = \begin{vmatrix} -4-t & 0 & -3 \\ 1 & 2-t & 5 \\ 0 & 0 & -4-t \end{vmatrix} = -(4+t) \begin{vmatrix} -(4+t) & 0 \\ 1 & 2-t \end{vmatrix}$$

$$= -(4+t)(-(4+t))(2-t) = (4+t)^2(2-t).$$

$$p_A(t) = (4+t)^2(2-t)$$

$$\lambda_1 = -4, \quad m_1 = 2$$

$$\lambda_2 = 2, \quad m_2 = 1.$$