Problem 5. Let $A = \begin{pmatrix} 3 & 6 & 3 & -9 & 3 \\ 3 & 6 & 1 & -1 & 2 \end{pmatrix}$

1. Find a basis for the nullspace Null(A).

$A = \begin{pmatrix} 3 & 6 & 3 & -9 & 3 \\ 3 & 6 & 1 & -1 & 2 \end{pmatrix} \to \begin{pmatrix} 1 & 2 & 1 & -3 & 1 \\ 0 & 0 & -2 & 8 & -1 \end{pmatrix} \Rightarrow \begin{pmatrix} 1 & 2 & 1 & -3 & 1 \\ 0 & 0 & 1 & -4 & \frac{1}{2} \end{pmatrix}$

$\to \begin{pmatrix} 1 & 2 & 0 & 1 & 1/2 \\ 0 & 0 & 1 & -4 & \frac{1}{2} \end{pmatrix}$ x_1, x_3 - basic

x_2, x_4, x_5 - free.

$x_5 = s$

$x_4 = t$

$x_3 = 4x_4 - \frac{1}{2}x_5 = 4t - \frac{1}{2}s$

$x_2 = u$

$x_1 = -2x_2 - x_4 - \frac{1}{2}x_5 = -2u - t - \frac{1}{2}s$

$\bar{x} = s\begin{pmatrix} -1/2 \\ 0 \\ -1/2 \\ 0 \\ 1 \end{pmatrix} + t\begin{pmatrix} -1 \\ 0 \\ 4 \\ 1 \\ 0 \end{pmatrix} + u\begin{pmatrix} -2 \\ 1 \\ 0 \\ 0 \\ 0 \end{pmatrix}$.

Basis for Null(A) is

$\left\{ \begin{pmatrix} -1/2 \\ 0 \\ -1/2 \\ 0 \\ 1 \end{pmatrix}, \begin{pmatrix} -1 \\ 0 \\ 4 \\ 1 \\ 0 \end{pmatrix}, \begin{pmatrix} -2 \\ 1 \\ 0 \\ 0 \\ 0 \end{pmatrix} \right\}$.

2. Find a basis the column space Col(A).

$\left\{ \begin{pmatrix} 3 \\ 3 \end{pmatrix}, \begin{pmatrix} 3 \\ 1 \end{pmatrix} \right\}$ is a basis for Col(A).

– pivot columns of A.

3. Find a basis for the row space Row(A).

$\{ (1, 2, 0, 1, \frac{1}{2}), (0, 0, 1, -4, \frac{1}{2}) \}$. – Nonzero rows of RREF(A).

4. Specify the nullity and the rank of matrix A and verify the dimension theorem.

nullity(A) = 3, rank(A) = 2

nullity(A) + rank(A) = 3 + 2 = 5 = # of columns of A.

5. Specify rank(A^T) and nullity(A^T) (Hint: for the latter, use the dimension theorem for A^T).

rank(A^T) = rank(A) = 2.

nullity(A^T) = (# of rows of A) – rank(A^T) = 2 – 2 = 0.