Test 2

Problem	1	2	3	4	5	6	Score
Points							

NAME: Solution Key

Show all your work for full credit.

Problem 1. Let P^2 be the vector space of quadratic polynomials ~~matrices~~, and let W be the subset of P^2 defined by

$$W = \left\{p \in P^2 :\ p(t) = 3a - 5b + (a+b)t + (2a-4b)t^2,\ \ a, b \in R\right\}.$$

1. Show that W is a subspace of P^2.

Let $p \in W$. Then $p(t) = 3a - 5b + (a+b)t + (2a-4b)t^2$

$$p(t) = a(3 + t + 2t^2) + b(-5 + t - 4t^2).$$

$\Rightarrow W = \text{span}\{\underbrace{3+t+2t^2}_{p_1(t)}, \underbrace{-5+t-4t^2}_{p_2(t)}\}$.

Since P^2 is a vector space, and $W = \text{span}\{p_1(t), p_2(t)\}$

W is a subspace of P^2.

Recall the theorem that the span of vectors in a vector space is a subspace.

2. Determine a basis for W and explain why it is a basis.

Let $\beta = \{3+t+2t^2, -5+t-4t^2\}$. Let $p_1(t) = 3+t+2t^2$, $p_2(t) = -5+t-4t^2$.

Note β spans W. Note $p_1(t) \neq c\,p_2(t)$ for any c, $\forall t$.

$\Rightarrow \beta$ is linearly independent.

Therefore, β is a basis for W.

3. What is the dimension of W? Explain. $\dim(W) = 2$ since β consists of 2 vectors.