Problem 4. Let $A = \begin{pmatrix} A_{11} & A_{12} \\ A_{21} & A_{22} \end{pmatrix}$ and $B = \begin{pmatrix} B_1 \\ B_2 \end{pmatrix}$, where

$$A_{11} = \begin{pmatrix} 2 & 1 \\ -1 & 0 \end{pmatrix}, \quad A_{12} = \begin{pmatrix} 4 \\ -2 \end{pmatrix}, \quad A_{21} = \begin{pmatrix} 0 & 0 \end{pmatrix}, \quad A_{22} = (-2) \quad B_1 = \begin{pmatrix} 3 \\ 1 \end{pmatrix}, \quad B_2 = (5).$$

1. Show that this block-partitioning of the matrices is consistent for the block multiplication of the product AB and find the product using block multiplication.

$$A = \begin{array}{c} \overbrace{}^{2}\ \overbrace{}^{1} \\ \left[\begin{array}{cc|c} 2 & 1 & 4 \\ -1 & 0 & -2 \\ \hline 0 & 0 & -2 \end{array}\right] \end{array}, \quad B = \left[\begin{array}{c} 3 \\ 1 \\ \hline 5 \end{array}\right] \begin{array}{l} \}2 \\ \}1 \end{array}$$

# of columns in A_{11} and A_{12} is the same as # of rows in B_1.
# of columns in A_{12} and A_{22} is the same as # of rows in B_2.

$$AB = \begin{pmatrix} A_{11} & A_{12} \\ A_{21} & A_{22} \end{pmatrix} \begin{pmatrix} B_1 \\ B_2 \end{pmatrix} = \begin{pmatrix} A_{11}B_1 + A_{12}B_2 \\ A_{21}B_1 + A_{22}B_2 \end{pmatrix}$$

$$A_{11}B_1 = \begin{pmatrix} 2 & 1 \\ -1 & 0 \end{pmatrix}\begin{pmatrix} 3 \\ 1 \end{pmatrix} = \begin{pmatrix} 6+1 \\ -3 \end{pmatrix} = \begin{pmatrix} 7 \\ -3 \end{pmatrix}$$

$$A_{12}B_2 = \begin{pmatrix} 4 \\ -2 \end{pmatrix} \cdot 5 = \begin{pmatrix} 20 \\ -10 \end{pmatrix}$$

$$A_{21}B_1 = (0,0)\begin{pmatrix} 3 \\ 1 \end{pmatrix} = 0$$

$$A_{22}B_2 = (-2) \cdot 5 = -10.$$

$$AB = \begin{pmatrix} \begin{pmatrix} 7 \\ -3 \end{pmatrix} + \begin{pmatrix} 20 \\ -10 \end{pmatrix} \\ 0 - 10 \end{pmatrix} = \begin{pmatrix} 27 \\ -13 \\ -10 \end{pmatrix}.$$

2. Is the product $B^T A^T$ defined? If yes, then find it.

$$B^T A^T = (AB)^T = \begin{pmatrix} 27 \\ -13 \\ -10 \end{pmatrix}^T = (27, -13, -10).$$