

Problem 3. Let $T(x) = Ax$ for $x \in \mathbb{R}^4$, where

$$A = \begin{pmatrix} 1 & 3 & -3 & 2 \\ 2 & 1 & -1 & 4 \\ 4 & 10 & -13 & 8 \\ 2 & 9 & -12 & 4 \end{pmatrix}.$$

Explain your answers.

1. Specify the domain and the co-domain of T .

Both ^{the} domain and the codomain of T are \mathbb{R}^4 .

2. Is T one-to-one?

$$\begin{pmatrix} 1 & 3 & -3 & 2 \\ 2 & 1 & -1 & 4 \\ 4 & 10 & -13 & 8 \\ 2 & 9 & -12 & 4 \end{pmatrix} \rightarrow \begin{pmatrix} 1 & 3 & -3 & 2 \\ 0 & -5 & 5 & 0 \\ 0 & -2 & -1 & 0 \\ 0 & 3 & -6 & 0 \end{pmatrix} \rightarrow \begin{pmatrix} 1 & 3 & -3 & 2 \\ 0 & 1 & -1 & 0 \\ 0 & 2 & 1 & 0 \\ 0 & 1 & -2 & 0 \end{pmatrix}$$

$$\rightarrow \begin{pmatrix} 1 & 3 & -3 & 2 \\ 0 & 1 & -1 & 0 \\ 0 & 0 & 3 & 0 \\ 0 & 0 & -1 & 0 \end{pmatrix} \rightarrow \begin{pmatrix} 1 & 3 & -3 & 2 \\ 0 & 1 & -1 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}$$

T is NOT one-to-one.

Equation $T(x) = 0$ has a nontrivial solution.

3. Is T onto? T is not onto. Columns of A do not span \mathbb{R}^4 .

4. Is T invertible? T is not invertible since it is not one-to-one.