

**Problem 2.** Let  $A = \begin{pmatrix} 3 & 2 & 0 \\ 5 & 3 & 0 \\ 0 & 0 & 1/4 \end{pmatrix}$ .

1. Find  $A^{-1}$ .

Method I

$$(A|I_3) = \left( \begin{array}{ccc|ccc} 3 & 2 & 0 & 1 & 0 & 0 \\ 5 & 3 & 0 & 0 & 1 & 0 \\ 0 & 0 & 1/4 & 0 & 0 & 1 \end{array} \right) \rightarrow \left( \begin{array}{ccc|ccc} 1 & 2/3 & 0 & 1/3 & 0 & 0 \\ 0 & -1/3 & 0 & -5/3 & 1 & 0 \\ 0 & 0 & 1/4 & 0 & 0 & 1 \end{array} \right)$$

$$\rightarrow \left( \begin{array}{ccc|ccc} 1 & 2/3 & 0 & 1/3 & 0 & 0 \\ 0 & 1 & 0 & 5 & -3 & 0 \\ 0 & 0 & 1/4 & 0 & 0 & 1 \end{array} \right) \rightarrow \left( \begin{array}{ccc|ccc} 1 & 0 & 0 & -3 & 2 & 0 \\ 0 & 1 & 0 & 5 & -3 & 0 \\ 0 & 0 & 1 & 0 & 0 & 4 \end{array} \right) \Rightarrow A^{-1} = \begin{pmatrix} -3 & 2 & 0 \\ 5 & -3 & 0 \\ 0 & 0 & 4 \end{pmatrix}$$

Method II

$$A^{-1} = \begin{pmatrix} \left( \begin{pmatrix} 3 & 2 \\ 5 & 3 \end{pmatrix}^{-1} & 0 \\ 0 & 0 & (1/4)^{-1} \end{pmatrix} = \begin{pmatrix} \begin{pmatrix} -3 & 2 \\ 5 & -3 \end{pmatrix} & 0 \\ 0 & 0 & 4 \end{pmatrix} = \begin{pmatrix} -3 & 2 & 0 \\ 5 & -3 & 0 \\ 0 & 0 & 4 \end{pmatrix}$$

$$\begin{pmatrix} 3 & 2 \\ 5 & 3 \end{pmatrix}^{-1} = \frac{1}{9-10} \begin{pmatrix} 3 & -2 \\ -5 & 3 \end{pmatrix} = \begin{pmatrix} -3 & 2 \\ 5 & -3 \end{pmatrix}$$

2. Using  $A^{-1}$ , solve the matrix equation:

$$AX = B, \text{ where } B = \begin{pmatrix} 1 & 2 \\ 2 & 1 \\ 1 & 2 \end{pmatrix}, \quad X = A^{-1}B$$

$$X = \begin{pmatrix} -3 & 2 & 0 \\ 5 & -3 & 0 \\ 0 & 0 & 4 \end{pmatrix} \begin{pmatrix} 1 & 2 \\ 2 & 1 \\ 1 & 2 \end{pmatrix} = \begin{pmatrix} 1 & -4 \\ -1 & 7 \\ 4 & 8 \end{pmatrix}.$$