Test 1

Problem	1	2	3	4	5	Grade
Points	/10	/10	/10	/10	/10	/50

NAME: Solution Key

Show all your work for a full credit. Calculators and crib sheets are not allowed.

Problem 1. Let $A = \begin{pmatrix} 2 & -4 & -1 & 1 & 6 \\ 1 & -2 & 3 & 4 & 10 \end{pmatrix}$ and $\mathbf{b} = \begin{pmatrix} -6 \\ -3 \end{pmatrix}$.

1. Solve the linear system $A\mathbf{x} = \mathbf{b}$ and write the solution in the vector-parametric form.

$$\begin{pmatrix} 2 & -4 & -1 & 1 & 6 & -6 \\ 1 & -2 & 3 & 4 & 10 & -3 \end{pmatrix} \to \begin{pmatrix} 1 & -2 & 3 & 4 & 10 & -3 \\ 2 & -4 & -1 & 1 & 6 & -6 \end{pmatrix} \to \begin{pmatrix} 1 & -2 & 3 & 4 & 10 & -3 \\ 0 & 0 & -7 & -7 & -14 & 0 \end{pmatrix}$$

$$\to \begin{pmatrix} 1 & -2 & 3 & 4 & 10 & -3 \\ 0 & 0 & 1 & 1 & 2 & 0 \end{pmatrix} \to \left(\begin{array}{ccccc|c} 1 & -2 & 0 & 1 & 4 & -3 \\ 0 & 0 & 1 & 1 & 2 & 0 \end{array}\right)$$

x_1, x_3 - basic variables, x_2, x_4, x_5 - free variables

$x_5 = t$

$x_4 = s$

$x_3 = -2x_5 - x_4 = -2t - s$

$x_2 = u$

$x_1 = -3 - 4x_5 - x_4 + 2x_2 =$

$= -3 - 4t - s + 2u$

$$x = \begin{pmatrix} -3 \\ 0 \\ 0 \\ 0 \\ 0 \end{pmatrix} + t\begin{pmatrix} -4 \\ 0 \\ -2 \\ 0 \\ 1 \end{pmatrix} + s\begin{pmatrix} -1 \\ 0 \\ -1 \\ 1 \\ 0 \end{pmatrix} + u\begin{pmatrix} 2 \\ 1 \\ 0 \\ 0 \\ 0 \end{pmatrix},$$

$t, s, u \in R$.

2. Find a general solution of the corresponding homogeneous linear system.

$$x_h = t\begin{pmatrix} -4 \\ 0 \\ -2 \\ 0 \\ 1 \end{pmatrix} + s\begin{pmatrix} -1 \\ 0 \\ -1 \\ 1 \\ 0 \end{pmatrix} + u\begin{pmatrix} 2 \\ 1 \\ 0 \\ 0 \\ 0 \end{pmatrix}, \quad t, s, u \in R.$$

3. Does the linear system $A\mathbf{x} = \mathbf{b}$ have a solution for any $\mathbf{b} \in R^2$? Explain.

Yes, it does. Every row of A has a pivot; therefore, columns of A span R^2.