

## Problem 5.

Let  $B = \left\{ \begin{bmatrix} 2 \\ -6 \end{bmatrix}, \begin{bmatrix} -1 \\ 4 \end{bmatrix} \right\}$  and  $C = \left\{ \begin{bmatrix} 4 \\ 5 \end{bmatrix}, \begin{bmatrix} 2 \\ 2 \end{bmatrix} \right\}$  be bases for  $R^2$ . Find the change of coordinate matrix from  $B$ -basis to  $C$ -basis and from  $C$ -basis to  $B$ -basis.

$$P_{E \leftarrow B} = \begin{bmatrix} 2 & -1 \\ -6 & 4 \end{bmatrix}, \quad P_{E \leftarrow C} = \begin{bmatrix} 4 & 2 \\ 5 & 2 \end{bmatrix}$$

$$P_{B \leftarrow E} = \begin{bmatrix} 2 & -1 \\ -6 & 4 \end{bmatrix}^{-1} = \frac{1}{8-6} \begin{bmatrix} 4 & 1 \\ 6 & 2 \end{bmatrix} = \begin{bmatrix} 2 & 1/2 \\ 3 & 1 \end{bmatrix}$$

$$P_{C \leftarrow E} = \begin{bmatrix} 4 & 2 \\ 5 & 2 \end{bmatrix}^{-1} = \frac{1}{8-10} \begin{bmatrix} 2 & -2 \\ -5 & 4 \end{bmatrix} = \begin{bmatrix} -1 & 1 \\ 5/2 & -2 \end{bmatrix}$$

$$\begin{aligned} P_{C \leftarrow B} &= P_{C \leftarrow E} P_{E \leftarrow B} = \begin{bmatrix} -1 & 1 \\ 5/2 & -2 \end{bmatrix} \begin{bmatrix} 2 & -1 \\ -6 & 4 \end{bmatrix} = \begin{bmatrix} -2-6 & 1+4 \\ 5+12 & -5/2-8 \end{bmatrix} \\ &= \begin{bmatrix} -8 & 5 \\ 17 & -21/2 \end{bmatrix} \end{aligned}$$

$$P_{B \leftarrow C} = P_{C \leftarrow B}^{-1} = \begin{bmatrix} -8 & 5 \\ 17 & -21/2 \end{bmatrix}^{-1} = \frac{1}{21 \cdot 4 - 17 \cdot 5} \begin{bmatrix} -21/2 & -5 \\ -17 & -8 \end{bmatrix}$$

$$= \begin{bmatrix} 21/2 & 5 \\ 17 & 8 \end{bmatrix}$$