Problem 2. Let $A = \begin{pmatrix} 2 & 2 & -2 & -4 & -4 \\ 2 & -1 & -5 & 5 & 17 \\ 1 & -1 & -3 & 4 & 12 \end{pmatrix}$

1. Find bases for the nullspace, the column space, and the row space of A.

Row Reduce A to RREF

$$\begin{pmatrix} 1 & -1 & -3 & 4 & 12 \\ 2 & 2 & -2 & -4 & -4 \\ 2 & -1 & -5 & 5 & 17 \end{pmatrix} \to \begin{pmatrix} 1 & -1 & -3 & 4 & 12 \\ 0 & 4 & 4 & -12 & -28 \\ 0 & 1 & 1 & -3 & -7 \end{pmatrix} \to \begin{pmatrix} 1 & -1 & -3 & 4 & 12 \\ 0 & 1 & 1 & -3 & -7 \\ 0 & 0 & 0 & 0 & 0 \end{pmatrix}$$

$$\to \begin{pmatrix} 1 & 0 & -2 & 1 & 5 \\ 0 & 1 & 1 & -3 & -7 \\ 0 & 0 & 0 & 0 & 0 \end{pmatrix}$$

x_3, x_4, x_5 - free

$$\begin{cases} x_5 = t \\ x_4 = s \\ x_3 = u \\ x_2 = -u + 3s + 7t \\ x_1 = 2u - s - 5t \end{cases} \Rightarrow x = t\begin{pmatrix} -5 \\ 7 \\ 0 \\ 0 \\ 1 \end{pmatrix} + s\begin{pmatrix} -1 \\ 3 \\ 0 \\ 1 \\ 0 \end{pmatrix} + u\begin{pmatrix} 2 \\ -1 \\ 1 \\ 0 \\ 0 \end{pmatrix}.$$

Basis for Null(A): $\left\{ \begin{pmatrix} -5 \\ 7 \\ 0 \\ 0 \\ 1 \end{pmatrix}, \begin{pmatrix} -1 \\ 3 \\ 0 \\ 1 \\ 0 \end{pmatrix}, \begin{pmatrix} 2 \\ -1 \\ 1 \\ 0 \\ 0 \end{pmatrix} \right\}$.

Basis for Col(A): $\left\{ \begin{pmatrix} 2 \\ 2 \\ 1 \end{pmatrix}, \begin{pmatrix} 2 \\ -1 \\ -1 \end{pmatrix} \right\}$ - pivot columns of A

Basis for Row(A):

$\{(1,0,-2,1,5), (0,1,1,-3,-7)\}$.

2. Specify the nullity and the rank of matrix A.

nullity(A) = 3

rank(A) = 2

3. Does a linear system $Ax = b$ have a solution for any $b \in R^3$? Explain.

No. The columns of A do not span R^3.

4. Suppose that the system $Ax = b$ has a solution for some $b \in R^3$. Is the solution unique?

No. There are free variables.