Test 2

Problem	1	2	3	4	5	Score
Points						

NAME: Solution Key

Show all your work for the full credit. Calculators and crib sheets are not allowed.

Problem 1. Let P_2 be the vector space of quadratic polynomials, and let

$$W = \left\{p(t) = at^2 + (a+2b)t + b) \mid a, b \in R\right\}.$$

1. Show that W is a subspace of P_2.

1) Take $a=b=0$. $p(t) = 0t^2+0t+0 = 0(t) \Rightarrow 0(t) \in W$.

2) $p_1(t) = a_1t^2 + (a_1+2b_1)t + b_1$, $p_2(t) = a_2t^2 + (a_2+2b_2)t + b_2$

$p_1(t)+p_2(t) = (a_1+a_2)t^2 + t(a_1+a_2+2(b_1+b_2)) + (b_1+b_2)$
Let $a_1+a_2 = \tilde{a}$ and $b_1+b_2 = \tilde{b}$. Then
$p_1(t)+p_2(t) = \tilde{a}t^2 + t(\tilde{a}+2\tilde{b}) + \tilde{b} \in W$.
Closed under addition

3) $p(t) = at^2 + (a+2b)t + b$, $\alpha \in R$

$\alpha p(t) = (\alpha a)t^2 + (\alpha a + 2(\alpha b))t + \alpha b$.
Let $\tilde{a} = \alpha a$ and $\tilde{b} = \alpha b$. Then $\alpha p(t) = \tilde{a}t^2 + (\tilde{a}+2\tilde{b})t + \tilde{b} \in W$.
Closed under scalar multiplication.
$\Rightarrow W$ is a subspace of P_2.

2. Determine a basis for W and explain why it is a basis.

$p(t) = at^2 + (a+2b)t + b = a(t^2+1) + b(2t+1)$.
Let $p_1(t) = t^2+1 \in W$ and $p_2(t) = 2t+1 \in W$
Basis for W: $B = \{t^2+1, 2t+1\}$.
B is a basis because $p_1(t)$ and $p_2(t)$ are linearly independent ($p_1(t) \neq cp_2(t)$) and B spans W, that is, $\forall p(t) \in W$,
$p(t) = ap_1(t) + bp_2(t)$.

3. What is the dimension of W? Explain. $\dim(W) = 2$ since B consists of 2 vectors.