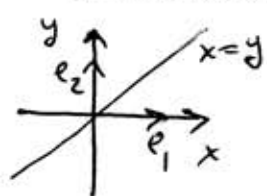


Problem 4. Let $T : \mathbb{R}^2 \rightarrow \mathbb{R}^2$ be a linear transformation that first reflects through the line $x = y$, then expands by a factor of -2 in the x -coordinate, and then reflects with respect to the y -axis.

1. Find the standard matrix of T .



1. Find the standard matrix of T . *reflect (x↔y), scale by (-2) in x-direction, reflects with respect y-axis*

$$e_1 = \begin{pmatrix} 1 \\ 0 \end{pmatrix} \rightarrow \begin{pmatrix} 0 \\ 1 \end{pmatrix} \rightarrow \begin{pmatrix} 0 \\ 1 \end{pmatrix} \rightarrow \begin{pmatrix} 0 \\ 1 \end{pmatrix}$$

$$e_2 = \begin{pmatrix} 0 \\ 1 \end{pmatrix} \rightarrow \begin{pmatrix} 1 \\ 0 \end{pmatrix} \rightarrow \begin{pmatrix} -2 \\ 0 \end{pmatrix} \rightarrow \begin{pmatrix} 2 \\ 0 \end{pmatrix}$$

$$[T] = [T(e_1), T(e_2)] = \begin{bmatrix} 0 & 2 \\ 1 & 0 \end{bmatrix}$$

2. Show that T is invertible and find the standard matrix of T^{-1}

T is invertible $\Leftrightarrow [T]$ is invertible.

$[T]$ is invertible since it is square and both columns are pivot.

$$[T]^{-1} = \frac{1}{-2} \begin{bmatrix} 0 & -2 \\ -1 & 0 \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ 1/2 & 0 \end{bmatrix} = [T]^{-1}$$

3. Find T^{-1} .

$$T^{-1} \begin{pmatrix} x \\ y \end{pmatrix} = [T]^{-1} \begin{pmatrix} x \\ y \end{pmatrix} = \begin{bmatrix} 0 & 1 \\ 1/2 & 0 \end{bmatrix} \begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} y \\ x/2 \end{pmatrix}$$

4. Find the image of vector $v = (1, 3)^T$ under transformation T .

$$T \begin{pmatrix} 1 \\ 3 \end{pmatrix} = \begin{pmatrix} 0 & 2 \\ 1 & 0 \end{pmatrix} \begin{pmatrix} 1 \\ 3 \end{pmatrix} = \begin{pmatrix} 6 \\ 1 \end{pmatrix}$$