

Problem 3. Let $T(x) = Ax$ for $x \in \mathbb{R}^4$, where

$$A = \begin{pmatrix} 1 & -1 & 4 & 0 \\ 2 & 0 & 1 & 3 \\ 3 & -2 & 3 & 6 \end{pmatrix}.$$

Explain your answers.

1. Is T a linear transformation?

Yes. $T(cx+y) = A(cx+y) = cAx + Ay = cT(x) + T(y),$
 $\forall x, y \in \mathbb{R}^4, \forall c \in \mathbb{R}.$

2. What is the domain of T ?

$$\mathbb{R}^4$$

3. What is the co-domain of T ?

$$\mathbb{R}^3$$

4. Is T one-to-one?

$$\begin{pmatrix} 1 & -1 & 4 & 0 \\ 2 & 0 & 1 & 3 \\ 3 & -2 & 3 & 6 \end{pmatrix} \rightarrow \begin{pmatrix} 1 & -1 & 4 & 0 \\ 0 & 2 & -7 & 3 \\ 0 & 1 & -9 & 6 \end{pmatrix} \rightarrow \begin{pmatrix} 1 & -1 & 4 & 0 \\ 0 & 1 & -9 & 6 \\ 0 & 2 & -7 & 3 \end{pmatrix} \rightarrow \begin{pmatrix} 1 & -1 & 4 & 0 \\ 0 & 1 & -9 & 6 \\ 0 & 0 & 11 & -9 \end{pmatrix}$$

No. A is 3×4 . A can't have a pivot in every column.

5. Is T onto?

Yes. A has a pivot in every row. Therefore, the columns of A span \mathbb{R}^3 .

6. Is T invertible?

No, A is not a square matrix.