

Problem 2. Let  $A = \begin{pmatrix} 1 & 2 & 3 \\ 0 & -1 & 2 \\ 1 & 1 & 1 \end{pmatrix}$ .

1. Find  $A^{-1}$ . Row reduce  $[A|I]$  to  $[I|A^{-1}]$ .

$$\begin{aligned} & \left( \begin{array}{ccc|ccc} 1 & 2 & 3 & 1 & 0 & 0 \\ 0 & -1 & 2 & 0 & 1 & 0 \\ 1 & 1 & 1 & 0 & 0 & 1 \end{array} \right) \rightarrow \left( \begin{array}{ccc|ccc} 1 & 2 & 3 & 1 & 0 & 0 \\ 0 & -1 & 2 & 0 & 1 & 0 \\ 0 & -1 & -2 & -1 & 0 & 1 \end{array} \right) \rightarrow \left( \begin{array}{ccc|ccc} 1 & 2 & 3 & 1 & 0 & 0 \\ 0 & -1 & 2 & 0 & 1 & 0 \\ 0 & 0 & -4 & -1 & -1 & 1 \end{array} \right) \\ & \rightarrow \left( \begin{array}{ccc|ccc} 1 & 2 & 3 & 1 & 0 & 0 \\ 0 & -1 & 2 & 0 & 1 & 0 \\ 0 & 0 & 1 & \frac{1}{4} & \frac{1}{4} & -\frac{1}{4} \end{array} \right) \rightarrow \left( \begin{array}{ccc|ccc} 1 & 2 & 0 & \frac{1}{4} & -\frac{3}{4} & \frac{3}{4} \\ 0 & -1 & 0 & \frac{1}{2} & -\frac{1}{2} & -\frac{1}{2} \\ 0 & 0 & 1 & \frac{1}{4} & \frac{1}{4} & -\frac{1}{4} \end{array} \right) \\ & \rightarrow \left( \begin{array}{ccc|ccc} 1 & 0 & 0 & -\frac{3}{4} & \frac{1}{4} & \frac{7}{4} \\ 0 & -1 & 0 & \frac{1}{2} & -\frac{1}{2} & -\frac{1}{2} \\ 0 & 0 & 1 & \frac{1}{4} & \frac{1}{4} & -\frac{1}{4} \end{array} \right) \end{aligned}$$

$$A^{-1} = \begin{pmatrix} -3/4 & 1/4 & 7/4 \\ 1/2 & -1/2 & -1/2 \\ 1/4 & 1/4 & -1/4 \end{pmatrix}.$$

2. Using  $A^{-1}$ , find the inverse of  $A^T$ .

$$(A^T)^{-1} = (A^{-1})^T = \begin{pmatrix} -3/4 & 1/2 & 1/4 \\ 1/4 & -1/2 & 1/4 \\ 7/4 & -1/2 & -1/4 \end{pmatrix}.$$