

Test 1

Problem	1	2	3	4	5	Grade
Points	/10	/10	/10	/10	/10	/50

NAME: Solution Key

Show all your work for a full credit. Calculators and crib sheets are not allowed.

Problem 1. Let $A = \begin{pmatrix} 2 & 2 & 4 & 0 \\ 1 & 2 & 4 & 0 \\ 3 & 0 & 6 & 6 \end{pmatrix}$ and $b = \begin{pmatrix} 22 \\ 12 \\ 24 \end{pmatrix}$.

1. Solve the linear system $Ax = b$ and write the solution in the parametric vector form.Row reduce $[A|b]$

$$\begin{pmatrix} 2 & 2 & 4 & 0 & 22 \\ 1 & 2 & 4 & 0 & 12 \\ 3 & 0 & 6 & 6 & 24 \end{pmatrix} \rightarrow \begin{pmatrix} 1 & 2 & 4 & 0 & 12 \\ 2 & 2 & 4 & 0 & 22 \\ 3 & 0 & 6 & 6 & 24 \end{pmatrix} \rightarrow \begin{pmatrix} 1 & 2 & 4 & 0 & 12 \\ 1 & 1 & 2 & 0 & 11 \\ 1 & 0 & 2 & 2 & 8 \end{pmatrix}$$

$$\rightarrow \begin{pmatrix} 1 & 2 & 4 & 0 & 12 \\ 0 & -1 & -2 & 0 & -1 \\ 0 & -2 & -2 & 2 & -4 \end{pmatrix} \rightarrow \begin{pmatrix} 1 & 2 & 4 & 0 & 12 \\ 0 & 1 & 2 & 0 & 1 \\ 0 & 1 & 1 & -1 & 2 \end{pmatrix} \rightarrow \begin{pmatrix} 1 & 2 & 4 & 0 & 12 \\ 0 & 1 & 2 & 0 & 1 \\ 0 & 0 & -1 & -1 & 1 \end{pmatrix}$$

$$\rightarrow \begin{pmatrix} 1 & 2 & 4 & 0 & 12 \\ 0 & 1 & 2 & 0 & 1 \\ 0 & 0 & 1 & 1 & -1 \end{pmatrix} \rightarrow \begin{pmatrix} 1 & 2 & 0 & -4 & 16 \\ 0 & 1 & 0 & -2 & 3 \\ 0 & 0 & 1 & 1 & -1 \end{pmatrix} \rightarrow \begin{pmatrix} 1 & 0 & 0 & 0 & 10 \\ 0 & 1 & 0 & -2 & 3 \\ 0 & 0 & 1 & 1 & -1 \end{pmatrix}$$

 x_4 - free, x_1, x_2, x_3 - basic x_1, x_2, x_3, x_4

$$\begin{cases} x_4 = t \\ x_3 = -1 - t \\ x_2 = 3 + 2t \\ x_1 = 10 \end{cases} \Rightarrow X = \begin{pmatrix} 10 \\ 3 \\ -1 \\ 0 \end{pmatrix} + t \begin{pmatrix} 0 \\ 2 \\ -1 \\ 1 \end{pmatrix}, t \in \mathbb{R}$$

2. Does the system have a unique solution? Explain.

No. The system has ∞ -many solutions since there is a free variable x_4 .3. Does the linear system have a solution for any $b \in \mathbb{R}^3$? Explain.Yes. There is a pivot position in every row of the matrix A .