

**Problem 3.** Let  $W$  be the subspace of  $\mathbb{R}^4$  spanned by the orthogonal vectors  $\left\{ \begin{bmatrix} 1 \\ 1 \\ 1 \\ 1 \end{bmatrix}, \begin{bmatrix} 1 \\ 1 \\ -1 \\ -1 \end{bmatrix}, \begin{bmatrix} 1 \\ -1 \\ 1 \\ -1 \end{bmatrix} \right\}$

Represent vector  $y = \begin{bmatrix} 4 \\ 3 \\ 2 \\ 1 \end{bmatrix}$  as the sum of the orthogonal projection of  $y$  on  $W$  and a vector in  $W^\perp$ .

Find  $\hat{y} = \text{proj}_W y = \frac{y \cdot v_1}{\|v_1\|^2} v_1 + \frac{y \cdot v_2}{\|v_2\|^2} v_2 + \frac{y \cdot v_3}{\|v_3\|^2} v_3$

$$\hat{y} = \frac{\begin{pmatrix} 4 \\ 3 \\ 2 \\ 1 \end{pmatrix} \cdot \begin{pmatrix} 1 \\ 1 \\ 1 \\ 1 \end{pmatrix}}{4} \begin{pmatrix} 1 \\ 1 \\ 1 \\ 1 \end{pmatrix} + \frac{\begin{pmatrix} 4 \\ 3 \\ 2 \\ 1 \end{pmatrix} \cdot \begin{pmatrix} 1 \\ 1 \\ -1 \\ -1 \end{pmatrix}}{4} \begin{pmatrix} 1 \\ 1 \\ -1 \\ -1 \end{pmatrix} + \frac{\begin{pmatrix} 4 \\ 3 \\ 2 \\ 1 \end{pmatrix} \cdot \begin{pmatrix} 1 \\ -1 \\ 1 \\ -1 \end{pmatrix}}{4} \begin{pmatrix} 1 \\ -1 \\ 1 \\ -1 \end{pmatrix}$$

$$= \frac{10}{4} \begin{pmatrix} 1 \\ 1 \\ 1 \\ 1 \end{pmatrix} + \begin{pmatrix} 1 \\ 1 \\ -1 \\ -1 \end{pmatrix} + \frac{1}{2} \begin{pmatrix} 1 \\ -1 \\ 1 \\ -1 \end{pmatrix} = \begin{pmatrix} 4 \\ 3 \\ 2 \\ 1 \end{pmatrix}.$$

$$z = y - \hat{y} = \begin{pmatrix} 4 \\ 3 \\ 2 \\ 1 \end{pmatrix} - \begin{pmatrix} 4 \\ 3 \\ 2 \\ 1 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \end{pmatrix}.$$

Vector  $y \in \text{span}(\{v_1, v_2, v_3\})$ .