# Final Test

| Problem | 1 | 2 | 3 | 4 | 5 | Score |
|---|---|---|---|---|---|---|
| Points | | | | | | |

**NAME:** Solution Key

**Show all your work for the full credit. Calculators and crib sheets are not allowed.**

**Problem 1.** 1. Show that $B = \left\{ \underset{u_1}{\begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}}, \underset{u_2}{\begin{bmatrix} 1 \\ 2 \\ -3 \end{bmatrix}}, \underset{u_3}{\begin{bmatrix} 5 \\ -4 \\ -1 \end{bmatrix}} \right\}$ is an orthogonal basis for $R^3$.

Let $B = \{u_1, u_2, u_3\}$.

$B$ is an orthogonal set if $u_i \cdot u_j = 0,\ i \neq j$

$u_1 \cdot u_2 = \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix} \cdot \begin{pmatrix} 1 \\ 2 \\ -3 \end{pmatrix} = 3 - 3 = 0$

$u_1 \cdot u_3 = \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix} \cdot \begin{pmatrix} 5 \\ -4 \\ -1 \end{pmatrix} = 5 - 5 = 0.$

$u_2 \cdot u_3 = \begin{pmatrix} 1 \\ 2 \\ -3 \end{pmatrix} \cdot \begin{pmatrix} 5 \\ -4 \\ -1 \end{pmatrix} = 5 - 8 + 3 = 0.$

Since $\{u_1, u_2, u_3\}$ is an orthogonal set of nonzero vectors, $B$ is an orthogonal basis for $R^3$.

2. Find the $B$-coordinate vector of vector $x = \begin{bmatrix} 1 \\ 5 \\ -7 \end{bmatrix}$ using the fact that basis $B$ is orthogonal.

Since $B$ is an orthogonal basis, $x = c_1 u_1 + c_2 u_2 + c_3 u_3$, where $c_i = \dfrac{x \cdot u_i}{\|u_i\|^2}$, $i = 1, 2, 3$. and $[x]_B = \begin{pmatrix} c_1 \\ c_2 \\ c_3 \end{pmatrix}$.

$c_1 = \dfrac{\begin{pmatrix} 1 \\ 5 \\ -7 \end{pmatrix} \cdot \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix}}{3} = -\dfrac{1}{3}$, $c_2 = \dfrac{\begin{pmatrix} 1 \\ 5 \\ -7 \end{pmatrix} \cdot \begin{pmatrix} 1 \\ 2 \\ -3 \end{pmatrix}}{14} = \dfrac{16}{7}$,

$c_3 = \dfrac{\begin{pmatrix} 1 \\ 5 \\ -7 \end{pmatrix} \cdot \begin{pmatrix} 5 \\ -4 \\ -1 \end{pmatrix}}{42} = -\dfrac{4}{21} \Rightarrow \boxed{[x]_B = \begin{pmatrix} -1/3 \\ 16/7 \\ -4/21 \end{pmatrix}}.$