Problem 4.

1. Show that the set $B = \left\{ \mathbf{b}_1 = \begin{pmatrix} 1 \\ -1 \\ 0 \end{pmatrix}, \mathbf{b}_2 = \begin{pmatrix} 1 \\ 1 \\ -2 \end{pmatrix}, \mathbf{b}_3 = \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix} \right\}$ is an orthogonal basis for R^3.

$$\left.\begin{array}{l} b_1 \cdot b_2 = 1 - 1 + 0 = 0 \\ b_1 \cdot b_3 = 1 - 1 + 0 = 0 \\ b_2 \cdot b_3 = 1 + 1 - 2 = 0 \end{array}\right\} b_i \cdot b_j = 0,\ i \neq j$$

$\Rightarrow$ B is an orthogonal set.

Since b_1, b_2, and b_3 are not zero $\Rightarrow$ B is an orthogonal basis for R^3.

2. Using orthogonality of B, find the B-coordinate vector of $y = \begin{pmatrix} 6 \\ 2 \\ 13 \end{pmatrix}$; that is, represent vector y as a linear combination of vectors in basis B.

$$y = \frac{b_1 \cdot y}{\|b_1\|^2} b_1 + \frac{b_2 \cdot y}{\|b_2\|^2} b_2 + \frac{b_3 \cdot y}{\|b_3\|^2} b_3$$

$$= \frac{6-2}{1+1}\begin{pmatrix} 1 \\ -1 \\ 0 \end{pmatrix} + \frac{6+2-26}{1+1+4}\begin{pmatrix} 1 \\ 1 \\ -2 \end{pmatrix} + \frac{6+2+13}{1+1+1}\begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix} \Rightarrow$$

$$y = 2b_1 - 3b_2 + 7b_3. \Rightarrow \boxed{[y]_B = \begin{pmatrix} 2 \\ -3 \\ 7 \end{pmatrix}}$$