Problem 2. Let $E = \left\{\begin{pmatrix}1\\0\end{pmatrix}, \begin{pmatrix}0\\1\end{pmatrix}\right\}$ be the standard basis for R^2, and let $B = \left\{\begin{pmatrix}1\\3\end{pmatrix}, \begin{pmatrix}2\\5\end{pmatrix}\right\}$ be another basis for R^2. Let T be a linear transformation on R^2 defined by

$$T(x, y) = (2y, 3x - y).$$

1. Find the change of coordinate matrix from the B-basis to the standard basis.

$$P_{E\leftarrow B} = \begin{pmatrix}1 & 2\\3 & 5\end{pmatrix} = [b_1, b_2].$$

2. Find the change of coordinate matrix from the standard basis to the B-basis.

$$P_{B\leftarrow E} = P_{E\leftarrow B}^{-1} = \begin{pmatrix}1 & 2\\3 & 5\end{pmatrix}^{-1} = \frac{1}{5-6}\begin{pmatrix}5 & -2\\-3 & 1\end{pmatrix} = \begin{pmatrix}-5 & 2\\3 & -1\end{pmatrix}.$$

3. Find the standard matrix $[T]_E$ of the linear transformation T.

$$[T]_E = [T(e_1), T(e_2)] = \begin{bmatrix}0 & 2\\3 & -1\end{bmatrix}$$

4. Using the relation $[T]_B = P_{B\leftarrow E}[T]_E P_{E\leftarrow B}$, find the matrix $[T]_B$ of the linear transformation T in the B-basis.

$$[T]_B = \begin{pmatrix}-5 & 2\\3 & -1\end{pmatrix}\begin{pmatrix}0 & 2\\3 & -1\end{pmatrix}\begin{pmatrix}1 & 2\\3 & 5\end{pmatrix} = \begin{pmatrix}6 & -12\\-3 & 7\end{pmatrix}\begin{pmatrix}1 & 2\\3 & 5\end{pmatrix} = \begin{pmatrix}-30 & -48\\18 & 29\end{pmatrix}.$$

$$\boxed{[T]_B = \begin{pmatrix}-30 & -48\\18 & 29\end{pmatrix}}$$