Test 3

Problem	1	2	3	4	Score
Points					

NAME: Solution Key

Show all your work for the full credit. Calculators and crib sheets are not allowed.

Problem 1. Let $T(x,y) = (x+y,\ -2x+4y)$ be a linear transformation on R^2. Find the B-basis for R^2 such that the matrix $[T]_B$ of T in the B-basis is diagonal, and find $[T]_B$.

Let $A = [T(e_1), T(e_2)] = \begin{bmatrix} 1 & 1 \\ -2 & 4 \end{bmatrix}$ - be the standard matrix of A.

Diagonalize A:

$P_A(t) = \det(A - tI) = \begin{vmatrix} 1-t & 1 \\ -2 & 4-t \end{vmatrix} = (1-t)(4-t)+2 = t^2 - 5t + 6 = 0$

$\lambda_1 = 2$, $\lambda_2 = 3$ - eigenvalues

A is diagonalizable, since it has 2 distinct eigenvalues and $n=2$.

$\boxed{[T]_B = \begin{bmatrix} 2 & 0 \\ 0 & 3 \end{bmatrix}.}$

Find bases for eigenspaces:

$A - \lambda_1 I = \begin{pmatrix} -1 & 1 \\ -2 & 2 \end{pmatrix} \to \begin{pmatrix} 1 & -1 \\ 0 & 0 \end{pmatrix} \Rightarrow v_1 = \begin{pmatrix} 1 \\ 1 \end{pmatrix}$

$A - \lambda_2 I = \begin{pmatrix} -2 & 1 \\ -2 & 1 \end{pmatrix} \to \begin{pmatrix} 1 & -1/2 \\ 0 & 0 \end{pmatrix} \Rightarrow v_2 = \begin{pmatrix} 1 \\ 2 \end{pmatrix}$

Take $\boxed{B = \left\{ \begin{pmatrix} 1 \\ 1 \end{pmatrix}, \begin{pmatrix} 1 \\ 2 \end{pmatrix} \right\}}$. In this basis $[T]_B$ is diagonal

$[T]_B = P^{-1}AP$, $P = P_{E \leftarrow B} = \begin{pmatrix} 1 & 1 \\ 1 & 2 \end{pmatrix}$.