

## Problem 2.

Let  $V = \mathbb{R}^2$  with bases  $B = \left\{ \begin{bmatrix} 4 \\ 7 \end{bmatrix}, \begin{bmatrix} 1 \\ 2 \end{bmatrix} \right\}$  and  $C = \left\{ \begin{bmatrix} 2 \\ 1 \end{bmatrix}, \begin{bmatrix} 5 \\ 2 \end{bmatrix} \right\}$ . Find the change of coordinate matrices from  $B$ -basis to  $C$ -basis and from  $C$ -basis to  $B$ -basis.

$B$ -to- $E$  matrix

$C$ -to- $E$  matrix

$$P_{E \leftarrow B} = \begin{bmatrix} 4 & 1 \\ 7 & 2 \end{bmatrix}, \quad P_{E \leftarrow C} = \begin{bmatrix} 2 & 5 \\ 1 & 2 \end{bmatrix}$$

$$P_{B \leftarrow E} = P_{E \leftarrow B}^{-1} = \begin{bmatrix} 4 & 1 \\ 7 & 2 \end{bmatrix}^{-1} = \frac{1}{8-7} \begin{bmatrix} 2 & -1 \\ -7 & 4 \end{bmatrix} = \begin{bmatrix} 2 & -1 \\ -7 & 4 \end{bmatrix}$$

$$P_{C \leftarrow E} = P_{E \leftarrow C}^{-1} = \begin{bmatrix} 2 & 5 \\ 1 & 2 \end{bmatrix}^{-1} = \frac{1}{4-5} \begin{bmatrix} 2 & -5 \\ -1 & 2 \end{bmatrix} = \begin{bmatrix} -2 & 5 \\ 1 & -2 \end{bmatrix}$$

$$P_{B \leftarrow C} = P_{B \leftarrow E} P_{E \leftarrow C} = \begin{bmatrix} 2 & -1 \\ -7 & 4 \end{bmatrix} \begin{bmatrix} 2 & 5 \\ 1 & 2 \end{bmatrix} = \begin{bmatrix} 4-1 & 10-2 \\ -14+4 & -35+8 \end{bmatrix} = \begin{bmatrix} 3 & 8 \\ -10 & -27 \end{bmatrix}$$

The change of coordinate matrix from  $C$ -basis to  $B$ -basis ↗

$$P_{C \leftarrow B} = P_{B \leftarrow C}^{-1} = \begin{bmatrix} 3 & 8 \\ -10 & -27 \end{bmatrix}^{-1} = \frac{1}{3(-27) - 8(-10)} \begin{bmatrix} -27 & -8 \\ 10 & 3 \end{bmatrix}$$

$$= \frac{1}{80-81} \begin{bmatrix} -27 & -8 \\ 10 & 3 \end{bmatrix} = \begin{bmatrix} 27 & 8 \\ -10 & -3 \end{bmatrix}$$

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The change of coord. matrix  
from  $B$ -basis to  $C$ -basis.