Test 2

Problem	1	2	3	4	5	Score
Points						

NAME: Solution Key

Show all your work for the full credit. Calculators and crib sheets are not allowed.

Problem 1. Let $V = R^{2\times 2}$ be a vector space of 2×2 matrices, and let W be a subset of V defined by

$$W = \left\{ A = \begin{pmatrix} 2a-3b & 0 \\ 0 & a+2b \end{pmatrix} \in V \mid a, b \in R \right\}.$$

1. Show that W is a subspace of V.

1) $0 \in W$? $0 = \begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix} \in W$, take $a = b = 0$.

2) Is W closed under addition? Let $A_1, A_2 \in W$.

$A_1 = \begin{pmatrix} 2a_1 - 3b_1 & 0 \\ 0 & a_1 + 2b_1 \end{pmatrix}$ and $A_2 = \begin{pmatrix} 2a_2 - 3b_2 & 0 \\ 0 & a_2 + 2b_2 \end{pmatrix}$

$$A_1 + A_2 = \begin{pmatrix} 2(a_1+a_2) - 3(b_1+b_2) & 0 \\ 0 & (a_1+a_2) + 2(b_1+b_2) \end{pmatrix} = \begin{pmatrix} 2\bar{a} - 3\bar{b} & 0 \\ 0 & \bar{a} + 2\bar{b} \end{pmatrix} \in W$$

where $\bar{a} = a_1 + a_2$ and $\bar{b} = b_1 + b_2 \Rightarrow A_1 + A_2 \in W$.

3) Is W closed under scalar multiplication?

$cA = \begin{pmatrix} 2(ca) - 3(cb) & 0 \\ 0 & ca + 2(cb) \end{pmatrix} \in W$, set $\bar{a} = ca$ and $\bar{b} = cb \Rightarrow W$ is a subspace of V.

2. Determine a basis for W and explain why it is a basis.

Take any $A = \begin{pmatrix} 2a-3b & 0 \\ 0 & a+2b \end{pmatrix} = \begin{pmatrix} 2a & 0 \\ 0 & a \end{pmatrix} + \begin{pmatrix} -3b & 0 \\ 0 & 2b \end{pmatrix}$

$= a\begin{pmatrix} 2 & 0 \\ 0 & 1 \end{pmatrix} + b\begin{pmatrix} -3 & 0 \\ 0 & 2 \end{pmatrix}.$

$\mathcal{B} = \left\{ \begin{pmatrix} 2 & 0 \\ 0 & 1 \end{pmatrix}, \begin{pmatrix} -3 & 0 \\ 0 & 2 \end{pmatrix} \right\}$ is a basis for W.

$\mathcal{B}$ is a basis since 1) $\mathcal{B}$ is linearly independent $\begin{pmatrix} 2 & 0 \\ 0 & 1 \end{pmatrix} \neq c\begin{pmatrix} -3 & 0 \\ 0 & 2 \end{pmatrix}$ $\forall c$.

2) $\mathcal{B}$ spans W: that is, $\forall A \in W$, is a linear combination of matrices in $\mathcal{B}$.

3. What is the dimension of W? Explain.

$\dim(W) = 2$ since $\mathcal{B}$ consists of two vectors.