

Problem 5. Let

$$A = \begin{bmatrix} 3 & 1 & 3 \\ 2 & 4 & -2 \\ -4 & 2 & -8 \end{bmatrix}.$$

1. Determine if the columns of matrix A span \mathbb{R}^3 ?

Row reduce A and see if \exists a pivot in every row.

$$\begin{bmatrix} 3 & 1 & 3 \\ 2 & 4 & -2 \\ -4 & 2 & -8 \end{bmatrix} \rightarrow \begin{bmatrix} 3 & 1 & 3 \\ 1 & 2 & -1 \\ -2 & 1 & -4 \end{bmatrix} \rightarrow \begin{bmatrix} 1 & 2 & -1 \\ 3 & 1 & 3 \\ -2 & 1 & -4 \end{bmatrix} \rightarrow \begin{bmatrix} 1 & 2 & -1 \\ 0 & -5 & 6 \\ 0 & 5 & -6 \end{bmatrix}$$

$$\rightarrow \begin{bmatrix} 1 & 2 & -1 \\ 0 & -5 & 6 \\ 0 & 0 & 0 \end{bmatrix}$$

There is no a pivot in Row 3.

Columns of A do not span \mathbb{R}^3

Since, for some $b \in \mathbb{R}^3$, the system $Ax = b$ is inconsistent.

2. Determine if the columns of matrix A are linearly independent?

Columns of A are linearly dependent.

The equation $Ax = 0$ has ∞ -many solutions

Since variable x_3 is free.