Problem 4. Let T be a linear transformation defined by $T(x_1, x_2) = \begin{pmatrix} x_1 + 2x_2 \\ 3x_1 + 4x_2 \\ 5x_1 + 6x_2 \end{pmatrix}$.

1. Specify the domain and the codomain of transformation T.

Domain is R^2 ((x_1, x_2) are variables)
Codomain is R^3 ($T(x_1, x_2) \in R^3$)

2. Is T one-to-one? Explain.

$[T] = \begin{bmatrix} 1 & 2 \\ 3 & 4 \\ 5 & 6 \end{bmatrix}$, $[T]$ is 3×2 ~~T is not one to one~~
Row reduce $[T]$ and check if $\exists$ pivot in every column.

$[T] \to \begin{bmatrix} 1 & 2 \\ 0 & -2 \\ 0 & -4 \end{bmatrix} \to \begin{bmatrix} 1 & 2 \\ 0 & 1 \\ 0 & 0 \end{bmatrix}$. Yes, T is one-to-one.
The equation $T(x) = 0$ has only the zero solution.

3. Is T onto? Explain.

T is not onto. $[T]$ is 3×2 and there is no pivot in the third row. is
The equation $T(x) = b$ ~~may be~~ inconsistent for some $b \in R^3$.

4. Is T a bijection? Explain.

T is not a bijection since T is not onto.

5. Is there T^{-1}? Explain. T is not invertible. because T is not onto.
(matrix $[T]$ is not square).