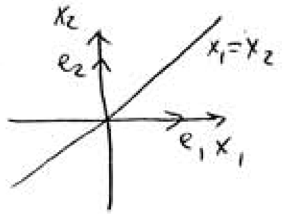
Problem 3. Let $T : R^2 \to R^2$ be a linear transformation that: 1) rotates through the angle of π radians counterclockwise, 2) extends by a factor of 2 in both directions, and 3) reflects with respect to the line $x_1 - x_2 = 0$.



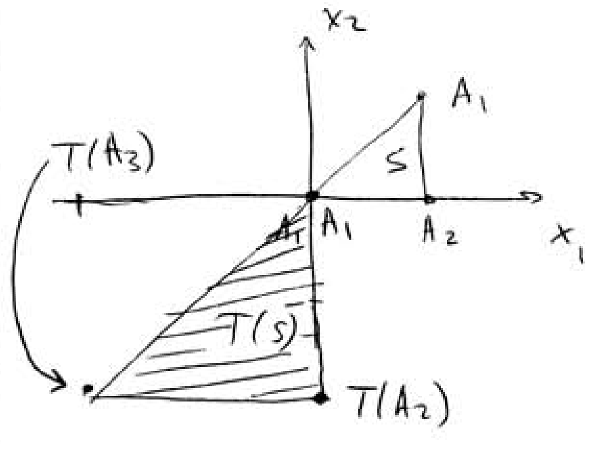
1. Find the standard matrix of the linear transformation.

$[T] = [T(e_1), T(e_2)]$, where $e_1 = \begin{pmatrix}1\\0\end{pmatrix}$, $e_2 = \begin{pmatrix}0\\1\end{pmatrix}$.

$$\left.\begin{array}{l} e_1 = \begin{pmatrix}1\\0\end{pmatrix} \to \begin{pmatrix}-1\\0\end{pmatrix} \to \begin{pmatrix}-2\\0\end{pmatrix} \to \begin{pmatrix}0\\-2\end{pmatrix} \\ e_2 = \begin{pmatrix}0\\1\end{pmatrix} \to \begin{pmatrix}0\\-1\end{pmatrix} \to \begin{pmatrix}0\\-2\end{pmatrix} \to \begin{pmatrix}-2\\0\end{pmatrix} \end{array}\right\} \Rightarrow [T] = \begin{bmatrix}0 & -2\\-2 & 0\end{bmatrix}.$$

2. Determine the image of the triangle S with vertices at $(0,0)^T$, $(1,0)^T$, and $(1,1)^T$ under the transformation T. Sketch a plot of the image $T(S)$ of the triangle.

$A_1 = \begin{pmatrix}0\\0\end{pmatrix}$, $A_2 = \begin{pmatrix}1\\0\end{pmatrix}$, $A_3 = \begin{pmatrix}1\\1\end{pmatrix}$



$$T(A_1) = \begin{bmatrix}0 & -2\\-2 & 0\end{bmatrix}\begin{pmatrix}0\\0\end{pmatrix} = \begin{pmatrix}0\\0\end{pmatrix}$$

$$T(A_2) = \begin{bmatrix}0 & -2\\-2 & 0\end{bmatrix}\begin{pmatrix}1\\0\end{pmatrix} = \begin{pmatrix}0\\-2\end{pmatrix}$$

$$T(A_3) = \begin{bmatrix}0 & -2\\-2 & 0\end{bmatrix}\begin{pmatrix}1\\1\end{pmatrix} = \begin{pmatrix}-2\\-2\end{pmatrix}$$