Problem 2. Let $A = \begin{pmatrix} 1 & -1 & 3 \\ 2 & -1 & 2 \\ 3 & 1 & -2 \end{pmatrix}$ and $B = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1/2 & 0 \\ 0 & 0 & 1/3 \end{pmatrix}$.

1. Find the inverses of A and B.

Row reduce $[A|I]$ to RREF.

$$\left[\begin{array}{ccc|ccc} 1 & -1 & 3 & 1 & 0 & 0 \\ 2 & -1 & 2 & 0 & 1 & 0 \\ 3 & 1 & -2 & 0 & 0 & 1 \end{array}\right] \to \left[\begin{array}{ccc|ccc} 1 & -1 & 3 & 1 & 0 & 0 \\ 0 & 1 & -4 & -2 & 1 & 0 \\ 0 & 4 & -11 & -3 & 0 & 1 \end{array}\right] \to \left[\begin{array}{ccc|ccc} 1 & -1 & 3 & 1 & 0 & 0 \\ 0 & 1 & -4 & -2 & 1 & 0 \\ 0 & 1 & -\frac{11}{4} & -\frac{3}{4} & 0 & \frac{1}{4} \end{array}\right]$$

$$\to \left[\begin{array}{ccc|ccc} 1 & -1 & 3 & 1 & 0 & 0 \\ 0 & 1 & -4 & -2 & 1 & 0 \\ 0 & 0 & \frac{5}{4} & \frac{5}{4} & -1 & \frac{1}{4} \end{array}\right] \to \left[\begin{array}{ccc|ccc} 1 & -1 & 3 & 1 & 0 & 0 \\ 0 & 1 & -4 & -2 & 1 & 0 \\ 0 & 0 & 1 & 1 & -\frac{4}{5} & \frac{1}{5} \end{array}\right] \to \left[\begin{array}{ccc|ccc} 1 & -1 & 0 & -2 & +12/5 & -3/5 \\ 0 & 1 & 0 & 2 & -\frac{11}{5} & 4/5 \\ 0 & 0 & 1 & 1 & -\frac{4}{5} & \frac{1}{5} \end{array}\right]$$

$$\to \left[\begin{array}{ccc|ccc} 1 & 0 & 0 & 0 & 1/5 & 1/5 \\ 0 & 1 & 0 & 2 & -11/5 & 4/5 \\ 0 & 0 & 1 & 1 & -\frac{4}{5} & \frac{1}{5} \end{array}\right] \Rightarrow A^{-1} = \begin{pmatrix} 0 & 1/5 & 1/5 \\ 2 & -11/5 & 4/5 \\ 1 & -4/5 & 1/5 \end{pmatrix}$$

$$B^{-1} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 3 \end{pmatrix} \text{ since } B^{-1}B = I_3.$$

2. Find the inverse of $(1/5)A$?

$$\left(\tfrac{1}{5}A\right)^{-1} = 5A^{-1} = 5\begin{pmatrix} 0 & 1/5 & 1/5 \\ 2 & -11/5 & 4/5 \\ 1 & -4/5 & 1/5 \end{pmatrix} = \begin{pmatrix} 0 & 1 & 1 \\ 10 & -11 & 4 \\ 5 & -4 & 1 \end{pmatrix}$$

3. Find the inverse of AB.

$$(AB)^{-1} = B^{-1}A^{-1} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 3 \end{pmatrix}\begin{pmatrix} 0 & 1/5 & 1/5 \\ 2 & -11/5 & 4/5 \\ 1 & -4/5 & 1/5 \end{pmatrix} = \begin{pmatrix} 0 & 1/5 & 1/5 \\ 4 & -22/5 & 8/5 \\ 3 & -12/5 & 3/5 \end{pmatrix}.$$