

Test 1

Problem	1	2	3	4	5	Grade
Points	/10	/10	/10	/10	/10	/50

NAME: Solution Key

Show all your work for a full credit. Calculators and crib sheets are not allowed.

Problem 1. 1. Solve the following linear system and write the general solution in the parametric vector form.

$$\begin{aligned} 2x_1 - 4x_2 + 16x_3 - 14x_4 &= 10 \\ -x_1 + 5x_2 - 17x_3 + 19x_4 &= -2 \\ x_1 - 3x_2 + 11x_3 - 11x_4 &= 4 \end{aligned} \quad [A|b] = \begin{bmatrix} 2 & -4 & 16 & -14 & 10 \\ -1 & 5 & -17 & 19 & -2 \\ 1 & -3 & 11 & -11 & 4 \end{bmatrix}$$

$$\rightarrow \begin{bmatrix} 1 & -2 & 8 & -7 & 5 \\ -1 & 5 & -17 & 19 & -2 \\ 1 & -3 & 11 & -11 & 4 \end{bmatrix} \rightarrow \begin{bmatrix} 1 & -2 & 8 & -7 & 5 \\ 0 & 3 & -9 & 12 & 3 \\ 0 & -1 & 3 & -4 & -1 \end{bmatrix} \rightarrow \begin{bmatrix} 1 & -2 & 8 & -7 & 5 \\ 0 & 1 & -3 & 4 & 1 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix} \leftarrow \text{REF}$$

Linear system is consistent, x_1, x_2 - basic variables
 x_3, x_4 - free variables

$$\rightarrow \begin{bmatrix} 1 & 0 & 2 & 1 & 7 \\ 0 & 1 & -3 & 4 & 1 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

$$\begin{cases} x_4 = s \\ x_3 = t \\ x_2 = 1 + 3t - 4s \\ x_1 = 7 - 2t - s \end{cases} \Rightarrow x = \begin{pmatrix} 7 \\ 1 \\ 0 \\ 0 \end{pmatrix} + t \begin{pmatrix} -2 \\ 3 \\ 1 \\ 0 \end{pmatrix} + s \begin{pmatrix} -1 \\ -4 \\ 0 \\ 1 \end{pmatrix}, t, s \in \mathbb{R}$$

2. Does the linear system have a solution for any $b \in \mathbb{R}^3$? Explain.

No. Third row does not have a pivot position. For some $b \in \mathbb{R}^3$, we might get the row $[0, 0, 0, 0 | \beta \neq 0]$ which corresponds to the equation $0 \cdot x_4 = \beta \neq 0$, which is false.

3. Determine a general solution of the corresponding homogeneous system.

$$x_h = t \begin{pmatrix} -2 \\ 3 \\ 1 \\ 0 \end{pmatrix} + s \begin{pmatrix} -1 \\ -4 \\ 0 \\ 1 \end{pmatrix}, s, t \in \mathbb{R}$$