

Problem 6. Let  $A = \begin{pmatrix} 3 & -1 & 4 \\ 1 & 2 & 0 \end{pmatrix}$ .

1. Compute  $A^T A$  using the column-row rule.

$$\begin{aligned} A^T A &= \begin{pmatrix} 3 & 1 \\ -1 & 2 \\ 4 & 0 \end{pmatrix} \begin{pmatrix} 3 & -1 & 4 \\ 1 & 2 & 0 \end{pmatrix} \\ &= \begin{pmatrix} 3 \\ -1 \\ 4 \end{pmatrix} (3, -1, 4) + \begin{pmatrix} 1 \\ 2 \\ 0 \end{pmatrix} (1, 2, 0) \\ &= \begin{bmatrix} 9 & -3 & 12 \\ -3 & 1 & -4 \\ 12 & -4 & 16 \end{bmatrix} + \begin{bmatrix} 1 & 2 & 0 \\ 2 & 4 & 0 \\ 0 & 0 & 0 \end{bmatrix} = \begin{bmatrix} 10 & -1 & 12 \\ -1 & 5 & -4 \\ 12 & -4 & 16 \end{bmatrix} \end{aligned}$$

2. Compute  $AA^T$  using the row-column rule.

$$\begin{aligned} \begin{pmatrix} 3 & -1 & 4 \\ 1 & 2 & 0 \end{pmatrix} \begin{pmatrix} 3 & 1 \\ -1 & 2 \\ 4 & 0 \end{pmatrix} &= \begin{pmatrix} 9+1+16 & 3-2 \\ 3-2 & 1+4 \end{pmatrix} = \\ &= \begin{pmatrix} 26 & 1 \\ 1 & 5 \end{pmatrix}. \end{aligned}$$