Problem 2. Determine if the vectors are linearly independent or linearly dependent. If the vectors are linearly dependent, express the third vector as a linear combination of the other two.

$$\begin{bmatrix} 1 \\ -2 \\ 1 \end{bmatrix}, \begin{bmatrix} 2 \\ 1 \\ -1 \end{bmatrix}, \begin{bmatrix} 7 \\ -4 \\ 1 \end{bmatrix} \rightarrow \begin{bmatrix} 1 & 2 & 7 \\ 0 & 5 & 10 \\ 0 & -3 & -6 \end{bmatrix} \rightarrow \begin{bmatrix} 1 & 2 & 7 \\ 0 & 1 & 2 \\ 0 & 0 & 0 \end{bmatrix} \rightarrow \begin{bmatrix} 1 & 0 & 3 \\ 0 & 1 & 2 \\ 0 & 0 & 0 \end{bmatrix}$$

3rd column is not pivot => vectors are linearly dependent.

x_1, x_2 - basic variables

x_3 - free variable

$$\begin{cases} x_1 = -3s \\ x_2 = -2s \\ x_3 = s \end{cases} \qquad x = s\begin{pmatrix} 3 \\ 2 \\ -1 \end{pmatrix}. \text{ Take } s = 1 \Rightarrow x = \begin{pmatrix} 3 \\ 2 \\ -1 \end{pmatrix}$$

$$3\begin{pmatrix} 1 \\ -2 \\ 1 \end{pmatrix} + 2\begin{pmatrix} 2 \\ 1 \\ -1 \end{pmatrix} - \begin{pmatrix} 7 \\ -4 \\ 1 \end{pmatrix} = 0 \Rightarrow \boxed{\begin{pmatrix} 7 \\ -4 \\ 1 \end{pmatrix} = 3\begin{pmatrix} 1 \\ -2 \\ 1 \end{pmatrix} + 2\begin{pmatrix} 2 \\ 1 \\ -1 \end{pmatrix}}$$