

## Test 1

Problem	1	2	3	4	5	6	Score
Points							

NAME: Solution Key

Show all your work for full credit. Calculators and crib sheets are not allowed.

Problem 1. Let  $A = \begin{pmatrix} 4 & 9 & 2 & 8 & 4 \\ 1 & 2 & -1 & 3 & 0 \end{pmatrix}$  and  $b = \begin{pmatrix} 4 \\ 2 \end{pmatrix}$ .1. Solve the linear system  $Ax = b$  and write the solution in the parametric vector form.

$$A = \begin{pmatrix} 4 & 9 & 2 & 8 & 4 \\ 1 & 2 & -1 & 3 & 0 \end{pmatrix} \rightarrow \begin{pmatrix} 1 & 2 & -1 & 3 & 0 & 2 \\ 4 & 9 & 2 & 8 & 4 & 4 \end{pmatrix} \rightarrow \begin{pmatrix} 1 & 2 & -1 & 3 & 0 & 2 \\ 0 & 1 & 6 & -4 & 4 & -4 \end{pmatrix}$$

$$\rightarrow \begin{pmatrix} 1 & 0 & -13 & 11 & -8 & 10 \\ 0 & 1 & 6 & -4 & 4 & -4 \end{pmatrix}$$

 $x_1, x_2$  - Basic variables $x_3, x_4, x_5$  - Free variables

$x_5 = t$

$x_4 = u$

$x_3 = v$

$x_2 = -4 - 6v + 4u - 4t$

$x_1 = 10 + 13v - 11u + 8t$

$$\begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \\ x_5 \end{bmatrix} = \begin{bmatrix} 10 \\ -4 \\ 0 \\ 0 \\ 0 \end{bmatrix} + v \begin{bmatrix} 13 \\ -6 \\ 1 \\ 0 \\ 0 \end{bmatrix} + u \begin{bmatrix} -11 \\ 4 \\ 0 \\ 1 \\ 0 \end{bmatrix} + t \begin{bmatrix} 8 \\ -4 \\ 0 \\ 0 \\ 1 \end{bmatrix}$$

$v, u, t \in \mathbb{R}$

2. Does the linear system have a solution for any  $b \in \mathbb{R}^2$ ? Explain.

Yes, it does. REF of  $A$  has a pivot position in every row  $\Rightarrow$  1st and 2nd columns of  $A$  span  $\mathbb{R}^2$ .