Quiz 4

Name: Solution Key

Let $A = \begin{pmatrix} 2 & 4 & -2 & 14 & 1 \\ 2 & 4 & 3 & -1 & -2 \\ 1 & 2 & -2 & 10 & 1 \\ -1 & -2 & 1 & -7 & 1 \end{pmatrix}$.

1. Find bases for the column space, the nullspace, and the row space of the matrix.

Row Reduce A to RREF.

$$A \to \begin{pmatrix} 1 & 2 & -2 & 10 & 1 \\ 2 & 4 & 3 & -1 & -2 \\ 2 & 4 & -2 & 14 & 1 \\ -1 & -2 & 1 & -7 & 1 \end{pmatrix} \begin{matrix} R_3 \\ \\ R_1 \\ \\ \end{matrix} \to \begin{pmatrix} 1 & 2 & -2 & 10 & 1 \\ 0 & 0 & 7 & -21 & -4 \\ 0 & 0 & 2 & -6 & -1 \\ 0 & 0 & -1 & 3 & 2 \end{pmatrix} \begin{matrix} \\ R_2-2R_1 \\ R_3-2R_1 \\ R_4+R_1 \end{matrix}$$

$$\to \begin{pmatrix} 1 & 2 & -2 & 10 & 1 \\ 0 & 0 & 1 & -3 & -2 \\ 0 & 0 & 2 & -6 & -1 \\ 0 & 0 & 7 & -21 & -4 \end{pmatrix} \begin{matrix} \\ R_4 \\ \\ R_2 \end{matrix} \to \begin{pmatrix} 1 & 2 & -2 & 10 & 1 \\ 0 & 0 & 1 & -3 & -2 \\ 0 & 0 & 0 & 0 & 3 \\ 0 & 0 & 0 & 0 & 10 \end{pmatrix} \to \begin{pmatrix} 1 & 2 & -2 & 10 & 1 \\ 0 & 0 & 1 & -3 & -2 \\ 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 \end{pmatrix}$$

$$\to \begin{pmatrix} 1 & 2 & -2 & 10 & 0 \\ 0 & 0 & 1 & -3 & 0 \\ 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 \end{pmatrix} \to \begin{pmatrix} 1 & 2 & 0 & 4 & 0 \\ 0 & 0 & 1 & -3 & 0 \\ 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 \end{pmatrix} \Leftarrow \text{RREF.}$$

Columns 1, 3, 5 are pivot $\Rightarrow$ Basis for $col(A)$ is $\left\{ \begin{pmatrix} 2 \\ 2 \\ 1 \\ -1 \end{pmatrix}, \begin{pmatrix} -2 \\ 3 \\ -2 \\ 1 \end{pmatrix}, \begin{pmatrix} 1 \\ -2 \\ 1 \\ 1 \end{pmatrix} \right\}$

Basis for $Row(A)$ is $\{(1,2,0,4,0), (0,0,1,-3,0), (0,0,0,0,1)\}$.

Solve $Ax=0$ (variables x_1, x_3, x_5 are basic; x_2, x_4 are free).

$$\begin{cases} x_5 = 0 \\ x_4 = t \\ x_3 = 3x_4 = 3t \\ x_2 = s \\ x_1 = -2s - 4t \end{cases} \Rightarrow \bar{x} = \begin{pmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{pmatrix} = s\begin{pmatrix} -2 \\ 1 \\ 0 \\ 0 \\ 0 \end{pmatrix} + t\begin{pmatrix} -4 \\ 0 \\ 3 \\ 1 \\ 0 \end{pmatrix}, \quad \forall s,t \in \mathbb{R}.$$

Basis for $N(A)$ is $\left\{ \begin{pmatrix} -2 \\ 1 \\ 0 \\ 0 \\ 0 \end{pmatrix}, \begin{pmatrix} -4 \\ 0 \\ 3 \\ 1 \\ 0 \end{pmatrix} \right\}$.

2. Specify the nullity and the rank of matrix A.

$\text{nullity}(A) = \dim(N(A)) = 2$. $\text{rank}(A) = \dim(Col(A)) = 3$.

3. Verify the dimension theorem for matrix A. ($n = 5$ – # of columns).

$\text{nullity}(A) + \text{rank}(A) = n \Rightarrow 2 + 3 = 5$.