Quiz 2

Name: Solution Key.

1. Determine if the following linear transformation is invertible, and, if it is invertible, then find the formula for the inverse transformation:

   $$T(x, y, z) = (x - y,\ x - z,\ -6x + 2y + 3z).$$

Find the standard matrix of T: $[T] = [T(e_1), T(e_2), T(e_3)]$

$$[T] = \begin{pmatrix} 1 & -1 & 0 \\ 1 & 0 & -1 \\ -6 & 2 & 3 \end{pmatrix}.$$ Row reduce $([T] \mid I)$

$$\left(\begin{array}{ccc|ccc} 1 & -1 & 0 & 1 & 0 & 0 \\ 1 & 0 & -1 & 0 & 1 & 0 \\ -6 & 2 & 3 & 0 & 0 & 1 \end{array}\right) \to \left(\begin{array}{ccc|ccc} 1 & -1 & 0 & 1 & 0 & 0 \\ 0 & 1 & -1 & -1 & 1 & 0 \\ 0 & -4 & 3 & 6 & 0 & 1 \end{array}\right) \to \left(\begin{array}{ccc|ccc} 1 & -1 & 0 & 1 & 0 & 0 \\ 0 & 1 & -1 & -1 & 1 & 0 \\ 0 & 0 & -1 & 2 & 4 & 1 \end{array}\right)$$

$[A]$ is invertible $\Rightarrow$ T is invertible

$$\to \left(\begin{array}{ccc|ccc} 1 & -1 & 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & -3 & -3 & -1 \\ 0 & 0 & 1 & -2 & -4 & -1 \end{array}\right) \to \left(\begin{array}{ccc|ccc} 1 & 0 & 0 & -2 & -3 & -1 \\ 0 & 1 & 0 & -3 & -3 & -1 \\ 0 & 0 & 1 & -2 & -4 & -1 \end{array}\right) \Rightarrow [T]^{-1} = \begin{pmatrix} -2 & -3 & -1 \\ -3 & -3 & -1 \\ -2 & -4 & -1 \end{pmatrix}.$$

$$T^{-1}(\bar{x}) = [T]^{-1}(\bar{x}) = (-2x - 3y - z,\ -3x - 3y - z,\ -2x - 4y - z).$$

Formula for the inverse transformation.

2. Use the column-row (outer product) form of matrix multiplication to compute the product AB if

   $$A = \begin{pmatrix} 1 & 2 \\ -2 & 1 \end{pmatrix} \quad \text{and} \quad B = \begin{pmatrix} 3 & -1 \\ 0 & 2 \end{pmatrix}.$$

$$AB = \begin{pmatrix} 1 \\ -2 \end{pmatrix}(3, -1) + \begin{pmatrix} 2 \\ 1 \end{pmatrix}(0, 2) = \begin{pmatrix} 3 & -1 \\ -6 & 2 \end{pmatrix} + \begin{pmatrix} 0 & 4 \\ 0 & 2 \end{pmatrix} = \begin{pmatrix} 3 & 3 \\ -6 & 4 \end{pmatrix}.$$