Quiz 5 Name: Solution Key

Diagonalize the matrix if possible:

1. $A = \begin{pmatrix} 1 & 1 & 1 \\ 0 & 1 & 0 \\ 0 & 1 & 0 \end{pmatrix}$

$$f_A(t) = \begin{vmatrix} 1-t & 1 & 1 \\ 0 & 1-t & 0 \\ 0 & 1 & -t \end{vmatrix} = (1-t)\begin{vmatrix} 1-t & 0 \\ 1 & -t \end{vmatrix} = (1-t)^2(-t) = 0$$

$\Rightarrow \lambda_1 = 0$ (mult. 1), $\lambda_2 = 1$ (mult. 2).

Consider $\lambda_2 = 1$.

$$A - \lambda_2 I = A - I = \begin{pmatrix} 0 & 1 & 1 \\ 0 & 0 & 0 \\ 0 & 1 & -1 \end{pmatrix} \to \begin{pmatrix} 0 & 1 & 1 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{pmatrix}$$

$\Rightarrow \dim(E_{\lambda_2}) = 1 < 2$ - multiplicity

$\Rightarrow$ A is <u>NOT</u> diagonalizable.

2. $A = \begin{pmatrix} 3 & -4 & 0 \\ 2 & -3 & 0 \\ 0 & 0 & 1 \end{pmatrix}$, $f_A(t) = \begin{vmatrix} 3-t & -4 & 0 \\ 2 & -3-t & 0 \\ 0 & 0 & 1-t \end{vmatrix} = (1-t)\begin{vmatrix} 3-t & -4 \\ 2 & -3-t \end{vmatrix}$

$$= (1-t)\left[-(9-t^2)+8\right] = (1-t)(t^2-1) = -(1-t)^2(1+t) = 0$$

$\Rightarrow \lambda_1 = -1$ (mult. 1), $\lambda_2 = 1$ (mult 2).

<u>Consider $\lambda_2 = 1$</u>

$$A - \lambda_2 I = \begin{pmatrix} 2 & -4 & 0 \\ 2 & -4 & 0 \\ 0 & 0 & 0 \end{pmatrix} \to \begin{pmatrix} 1 & -2 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} \Rightarrow \begin{cases} x_3 = t \\ x_2 = s \\ x_1 = 2s \end{cases} \Rightarrow x = s\begin{pmatrix} 2 \\ 1 \\ 0 \end{pmatrix} + t\Big($$

$\dim(E_{\lambda_2}) = 2 =$ alg. multiplicity $\Rightarrow$ diagonalizable.

Consider $\lambda_1 = -1$

$$A + I = \begin{pmatrix} 4 & -4 & 0 \\ 2 & -2 & 0 \\ 0 & 0 & 2 \end{pmatrix} \to \begin{pmatrix} 1 & -1 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{pmatrix} \Rightarrow \begin{cases} x_3 = 0 \\ x_2 = t \\ x_1 = -t \end{cases} \Rightarrow \bar{x} = t\begin{pmatrix} -1 \\ 1 \\ 0 \end{pmatrix}.$$

$A = PDP^{-1}$, where $P = \begin{pmatrix} 2 & 0 & -1 \\ 1 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix}$ and $D = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & -1 \end{pmatrix}$.