Quiz 4 Name: Solution Key

1. Determine if the set $W = \{p \in P_3 |\ p' = 2p\}$ is a subspace of P_3, where p' is the derivative of p.

P_3 is a vector space and W is a subset of P_3.

Let $\theta(t)$ be the zero polynomial in P_3: $\theta(t)=0, \forall t \in \mathbb{R}$.

1) $\theta'(t) = 0 = 2\theta(t), \forall t \in \mathbb{R} \Rightarrow \theta \in W$.

2) Let $p_1, p_2 \in W$: $p_i' = 2p_i$, $i=1,2$.

$(p_1+p_2)' = p_1' + p_2' = 2p_1 + 2p_2 = 2(p_1+p_2) \Rightarrow p_1+p_2 \in W$ (closed under vector addition).

3) $\forall c \in \mathbb{R}, \forall p \in W$,

$(cp)' = cp' = c2p = 2(cp) \Rightarrow cp \in W$. (Closed under sc. multipl.)

$\Rightarrow W$ is a subspace of P_3.

2. Find a basis for the nullspace of the matrix $\begin{pmatrix} 1 & 2 & 0 & -1 & 2 \\ -2 & -4 & 3 & 5 & -1 \\ 3 & 6 & 3 & 5 & -1 \end{pmatrix} = A$. Solve $Ax=0$.

Row reduce A.

$$A \to \begin{pmatrix} 1 & 2 & 0 & -1 & 2 \\ 0 & 0 & 3 & 3 & 3 \\ 0 & 0 & 3 & 8 & -7 \end{pmatrix} \to \begin{pmatrix} 1 & 2 & 0 & -1 & 2 \\ 0 & 0 & 1 & 1 & 1 \\ 0 & 0 & 3 & 8 & -7 \end{pmatrix} \to \begin{pmatrix} 1 & 2 & 0 & -1 & 2 \\ 0 & 0 & 1 & 1 & 1 \\ 0 & 0 & 0 & 5 & -10 \end{pmatrix}$$

$$\to \begin{pmatrix} 1 & 2 & 0 & -1 & 2 \\ 0 & 0 & 1 & 1 & 1 \\ 0 & 0 & 0 & 1 & -2 \end{pmatrix} \to \begin{pmatrix} 1 & 2 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 3 \\ 0 & 0 & 0 & 1 & -2 \end{pmatrix} \text{ (RREF).}$$

x_1, x_3, x_4 – basic, x_2, x_5 – free

$$\begin{cases} x_5 = s \\ x_4 = 2s \\ x_3 = -3s \\ x_2 = t \\ x_1 = -2t \end{cases} \Rightarrow \bar{x} = t\begin{pmatrix} -2 \\ 1 \\ 0 \\ 0 \\ 0 \end{pmatrix} + s\begin{pmatrix} 0 \\ 0 \\ -3 \\ 2 \\ 1 \end{pmatrix}, \ t,s \in \mathbb{R} \Rightarrow \boxed{B = \left\{ \begin{pmatrix} -2 \\ 1 \\ 0 \\ 0 \\ 0 \end{pmatrix}, \begin{pmatrix} 0 \\ 0 \\ -3 \\ 2 \\ 1 \end{pmatrix} \right\}}.$$

3. Determine if the set $\{t^2 + 2t + 3,\ -t^2 + 2,\ t^2 + 4t + 5\}$ is linearly dependent in P_2.

$c_1(t^2+2t+3) + c_2(-t^2+2) + c_3(t^2+4t+5) = 0$

$t^2(c_1 - c_2 + c_3) + t(2c_1 + 4c_3) + (3c_1 + 2c_2 + 5c_3) = 0$.

$$\begin{cases} c_1 - c_2 + c_3 = 0 \\ 2c_1 + 4c_3 = 0 \\ 3c_1 + 2c_2 + 5c_3 = 0 \end{cases} \Rightarrow \begin{pmatrix} 1 & -1 & 1 \\ 2 & 0 & 4 \\ 3 & 2 & 5 \end{pmatrix} \to \begin{pmatrix} 1 & -1 & 1 \\ 0 & 2 & 2 \\ 0 & 5 & 2 \end{pmatrix} \to \begin{pmatrix} 1 & -1 & 1 \\ 0 & 1 & 1 \\ 0 & 0 & 1 \end{pmatrix}$$

All variables are basic $\Rightarrow A\bar{c} = 0 \Rightarrow \bar{c} = 0 \Rightarrow c_1 = c_2 = c_3 = 0$.

The set is linearly independent.